STUDYING SINGULARITIES THROUGH NUMERICAL MODELS

I. Approach to the singularity

- behavior near the singularity
- Gowdy spacetimes
- more general spacetimes

II. Critical gravitational collapse

- Choptuik's discovery
- explanation of scaling
- what remains to be done

behavior near the singularity

Singularity theorems tell us
that singularities occur
in a wide range of circumstances

But they give us
very little information about
the nature of the singularity

We expect that tidal forces blow up

Some terms in Einsetein's equations
blow up

therefore other terms might be
by comparison negligible

the singularity might be relatively simple

Gowdy spacetimes

$$ds^2 = e^{(t-\lambda)/2}[-e^{-2t}dt^2 + dx^2]$$

$$+e^{-t}\left[e^P(dy + Q dz)^2 + e^{-P}dz^2\right]$$

Vacuum Einstein equations

$$P_{tt} - e^{-2t}P_{xx} - e^{2P}(Q_t^2 - e^{-2t}Q_x^2) = 0$$

$$Q_{tt} - e^{-2t}Q_{xx} + 2(P_tQ_t - e^{-2t}P_xQ_x) = 0$$

numerical methods

evaluate spatial derivatives
using centered differences

$$F_x \rightarrow \frac{F_{i+1} - F_{i-1}}{2\Delta x}$$

$$F_{xx} \rightarrow \frac{F_{i+1} + F_{i-1} - 2F_i}{(\Delta x)^2}$$

Evolve in time using 3 step ICN
$S_t = W$ implemented as

$$S^{n+1} = S^n + \frac{\Delta t}{2}[W(S^n) + W(S^{n+1})]$$

results

Numerical simulations
(Berger, Moncrief, . . .)
$P \to v(x)t$ and $Q \to Q(x)$ as $t \to \infty$
(but spikes at isolated points)

Global results
(Isenberg, Moncrief, Chrusciel)

Local, near singularity results
(Rendall, Kichenassamy)

more general spacetimes

U(1) spacetimes

Numerical simulations
(Berger, Moncrief)
Local Mixmaster behavior

No symmetry

Local, near singularity result
for Einstein-scalar equations
(Rendall, Andersson)
Local Kasner

Numerical simulations
(Garfinkle, Miller, Berger, Duncan)
work in progress

Critical gravitational collapse

Choptuik's discovery

collapse of a spherically symmetric scalar field ϕ initial data $\phi_p(0, r)$

for $p > p*$ a black hole forms

for $p < p*$ the field disperses

scaling of black hole mass:
for p near $p*$

$$M = c(p - p*)^\gamma$$

$$\gamma \approx .374$$

critical solution

the $p = p*$ solution
is discretely self-similar

After a certain amount of time
the profile of the scalar field
repeats itself with the scale
of space shrunk.

There is a diffeomorphism ζ such that

$$\zeta^*(g_{ab}) = e^{-2\Delta} g_{ab}$$

$$\zeta^*(\phi) = \phi$$

$$\Delta \approx 3.45$$

cases of critical collapse

many types of matter
in spherical symmetry

vacuum axisymmetry
(Abrahams and Evans)

no symmetry (perturbative)
(Gundlach and Martin-Garcia)

numerical methods

adaptive mesh refinement
(robust but difficult)

characteristic method
(simpler)

explanation of scaling

mass scaling can be explained
(Koike, Hara, Adachi,
Gundlach, Hod, Piran)
by assuming a DSS critical solution
with one unstable mode $\propto e^{\kappa T}$

Then the mass scaling exponent is
$\gamma = 1/\kappa$

The argument used also implies
scaling of other quantities
and on both sides of the critical solution
(Garfinkle, Duncan)

This suggests critical collapse
occurs in other systems
(Isenberg, Liebling, Bizon)

what remains to be done

numerical studies of critical collapse
in axisymmetry and no symmetry

investigation of critical behavior
in other hyperbolic
(and parabolic) systems

better understanding of
the dynamical systems picture