Open Problems from General Relativistic Astrophysics

John L. Friedman

Cargese 2002

Outline

I. Symmetry of perfect-fluid equilibria:
Static stars are spherical

II. Existence of stationary, asymptotically flat, nonaxisymmetric stars

III. Linearization stability of the stationary Einstein-Euler equations

IV. Existence of a class of stationary radiative solutions:
Binary systems with helical Killing vectors.

I. STATIC STARS SPHERICAL

SKETCH OF PROOF FOR BROAD CLASS OF REALISTIC EQUATIONS OF STATE

A.

SPHERICALLY SYMMETRIC $\Leftrightarrow$ SPATIALLY CONFORMALLY FLAT

AVEZ
KUNZLE
LINDBLOM

$$g_{\text{SPHSYMM}} = -N^2 dt^2 + \Omega^2 \delta \quad \sim \text{FLAT 3-METRIC}$$

OUTSIDE STAR

$$N = \frac{1 - M/2r}{1 + M/2r} \qquad \Omega = (1 + M/2r)^2$$

$$\Omega = \Omega(N) \quad \text{EVERYWHERE}$$

B.

STATIC METRIC HAS FORM

$$g = -N^2 dt^2 + {}^3g$$

THEN g SPHERICAL $\Leftrightarrow$ $\hat{g} := \Omega^{-2}(N)\cdot {}^3g = \delta$

$\Omega(N) = \Omega_{\text{SPH-SYMM}}(N)$ FOR MASS M OF g

C.

$\hat{g}$ HAS $\hat{M} = 0$

BY POSITIVE MASS TM

$\hat{R}$ NON-NEGATIVE $\Rightarrow$ $\hat{g} = \delta$ MASOOD-UL-ALAM

$\Rightarrow$ UNIFORM DENSITY STARS SPHERICAL & LINDBLOM

$\Rightarrow$ STARS WITH $p = p(\epsilon)$ SPHERICAL IF

$$\gamma \frac{1 + 3\frac{p}{\epsilon} - 6\frac{p^2}{\epsilon^2}}{(1+\frac{p}{\epsilon})(1+3\frac{p}{\epsilon})} + \frac{d\gamma}{dp/p} \geq \frac{6}{5} + \frac{8}{5}\frac{\frac{p}{\epsilon}}{1+3\frac{p}{\epsilon}}, \qquad \gamma := \frac{d\log p}{d\log \rho} = \frac{dp}{d\epsilon}\frac{\epsilon+p}{p}$$

II. $\exists$ STATIONARY NONAXISYMMETRIC PERFECT-FLUID STARS ?

NEWTONIAN APPROXIMATION:

DEDEKIND ELLIPSOID (UNIF DENSITY)

ANALOGUES FOR $p = p(\epsilon)$

STATIONARY FLOW IN INERTIAL FRAME SUGGESTS MAY EXIST IN EXACT THEORY - NO RADIATION

STRONGER ARGUMENT FROM LINEARIZED GR

PERTURB ROTATING, AXISYMMETRIC STAR

$$\delta G_{\alpha\beta} = 8\pi\, \delta T_{\alpha\beta} \qquad \overset{\text{BACKGROUND}}{\mathring{G}_{\alpha\beta} = 8\pi \mathring{T}_{\alpha\beta}}$$

FIND INSTABILITY TO NONAXISYMMETRIC PERTURBATION SETS IN THROUGH TIME-INDEPENDENT SOLUTION TO LINEARIZED EQS ("ZERO-FREQUENCY MODE")

IS THERE A FAMILY OF EXACT SOLUTIONS TO THE TIME-INDEPENDENT EINSTEIN-EULER SYSTEM TANGENT TO EACH TIME-INDEPENDENT PERT OF STATIONARY, AXISYMMETRIC ROTATING STAR? $\quad u^\alpha = \gamma(t^\alpha + \Omega \varphi^\alpha)$

NOT YET ANSWERED IN NEWTONIAN APPROXIMATION

$$\nabla^2 \delta\Phi = 4\pi\, \delta\rho \qquad \overset{?}{\Rightarrow} \qquad \text{FAMILY} \quad \nabla^2 \Phi_\lambda = 4\pi \rho_\lambda$$

$$\delta\left[v \cdot \nabla v + \frac{\nabla p}{\rho} + \nabla\Phi\right] = 0 \qquad\qquad v_\lambda \cdot \nabla v_\lambda + \frac{\nabla p_\lambda}{\rho_\lambda} + \nabla\Phi_\lambda = 0$$

COMPRESSIBLE DEDEKIND MODELS : URYU, ERIGUCHI

NO EXISTENCE PROOF

NO NUMERICAL CONSTRUCTION YET IN GR

A BIFURCATION TM $\Rightarrow$ LARGE FAMILY OF STATIONARY PERFECT-FLUID MODELS WITH SURFACE HAVING ANGULAR DEPENDENCE $\approx \cos m\phi$, ALL m

EACH A SPACETIME W/ A SINGLE TIMELIKE KV + NO OTHER SYMMETRY VECTOR.

ANGULAR VELOCITY Ω_c AT WHICH INSTABILITY SETS IN AS FUNCTION OF CENTRAL DENSITY ϵ_c

MODES WITH ANGULAR DEPENDENCE $e^{im\phi}$

MODE UNSTABLE $\Omega > \Omega_c$

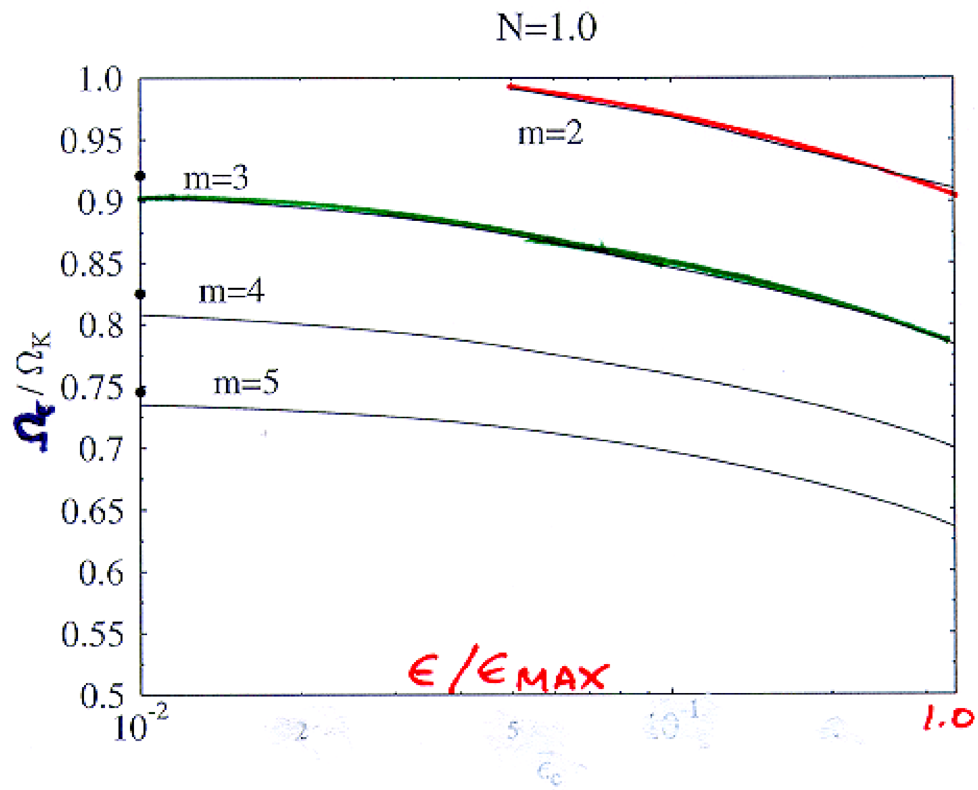
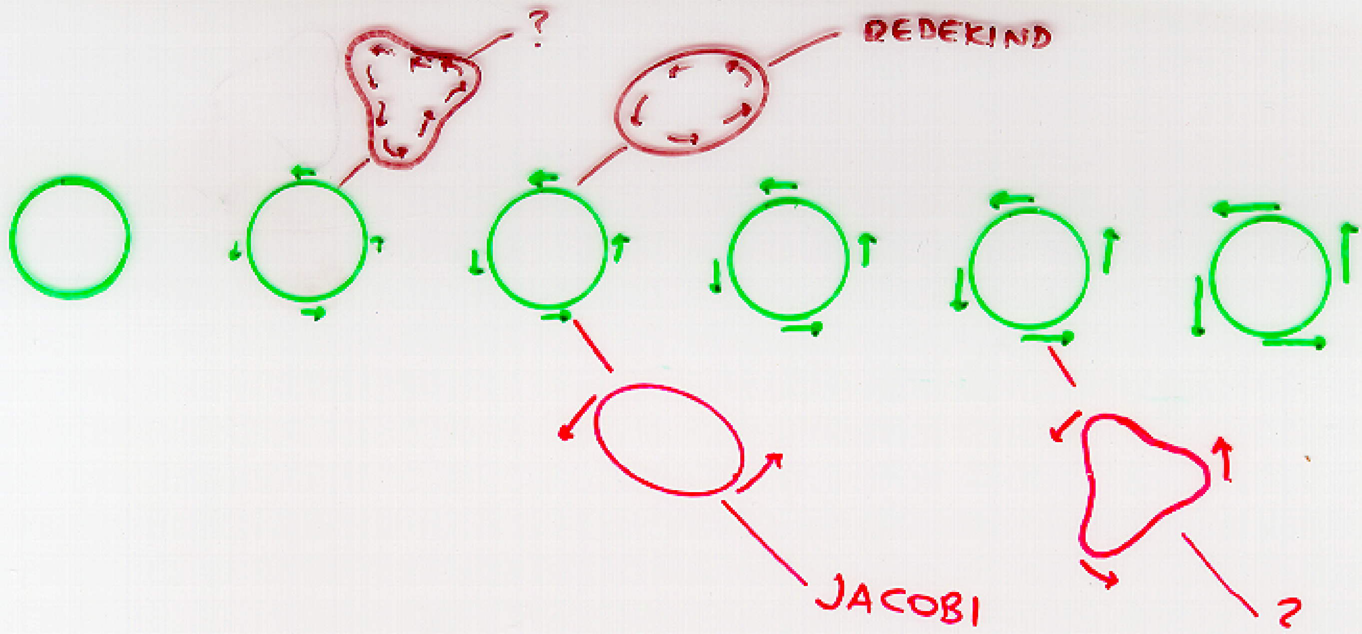


FIG. 2. Critical ratio of angular velocity to Kepler velocity vs. a dimensionless central energy density $\bar{\epsilon}_c$ for the $m = 2, 3, 4$ and 5 neutral modes of $n = 1.0$ polytropes. The largest value of $\bar{\epsilon}_c$ shown corresponds to the most relativistic stable configurations, while the lowest $\bar{\epsilon}_c$ corresponds to less relativistic configurations. The filled circles on the vertical axis represent the Newtonian limit.

STERGIOULAS, MORSINK, BLATTNIG

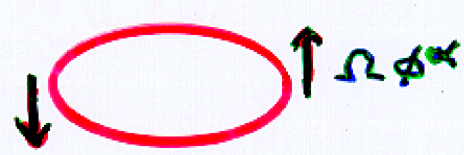
BIFURCATIONS ALONG THE MACLAURIN SEQUENCE



III. JACOBI ELLIPSOIDS, BINARIES, + HELICAL KILLING VECTOR

YOUNG, RAPIDLY ROTATING NSs ARE LIKELY TO BE UNSTABLE TO A DEDEKIND-LIKE MODE ($m \geq 2$). IF SETTLE DOWN TO STATIONARY NONAXISYMM STATE, CRUST MAY FREEZE IN THE ASYMMETRY, LEADING (AFTER, SAY, MAGNETIC PINNING ENFORCES UNIFORM ROTATION) TO PRECESSING, NONAXISYMM STAR.

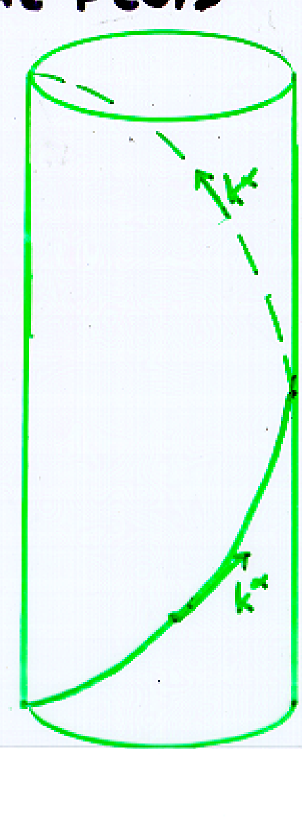
MORE INTENSIVE NUMERICAL WORK ON AN INSTABILITY, DURING FORMATION, TO JACOBI-LIKE CONFIGURATION



IN NEWTONIAN APPROXIMATION, AGAIN HAVE SINGLE SYMMETRY VECTOR, THIS TIME ALONG THE FLUID 4-VELOCITY

$$k^\alpha = t^\alpha + \Omega\phi^\alpha$$

$$\vec{\phi} = \hat{z} \times \vec{r} = x\hat{y} - y\hat{x}$$



IN EXACT THEORY TO MAINTAIN

HELICAL SYMMETRY

MUST HAVE EQUAL AMOUNTS OF INGOING + OUTGOING RADIATION

PRESENT DATA SETS FOR BINARY NEUTRON STARS + BHS ARE BASED ON IWM APPROXIMATION (ISENBERG-WILSON-MATHEWS)

5 METRIC FUNCTIONS LAPSE N, SHIFT β^a, CONFORMAL FACTOR ϕ

SPATIALLY CONFORMALLY FLAT

5 FIELD EQS

$$\mathcal{H} = \rho$$
$$\mathcal{H}^a = j^a$$
$${}^3g^{ab} G_{ab} = 8\pi {}^3g^{ab} T_{ab}$$

- RADIATION CONTENT UNCONTROLLED
- $F = ma \Rightarrow \Omega = \Omega(r)$ NOT ENFORCED

$\Rightarrow$ ORBIT ELLIPTICAL, NOT CIRCULAR

SEVERAL GROUPS NOW WORKING ON ADDING ADDITIONAL DEGREE OF FREEDOM

(6 INDEPENDENT METRIC COMPONENTS AFTER GAUGE CHOICE)

AND SATISFYING ADDITIONAL EQ.

PRICE, BROMLEY, BEETLE
GOURGOULHON..

URYU, SHIBATA, TSOKAROS, JF

TO OBTAIN NUMERICAL SOL'N FOR BINARY W/ HELICAL KV.

FAR FROM SOURCE,

$R \gtrsim 10{,}000\,M$,

E(RADIATION ENCLOSED) ~ SOURCE MASS M

BUT FOR NUMERICAL GRID

$E_{RADIATION} < \frac{1}{100} M$

$\Rightarrow$ SEE APPROXIMATE ASYMPTOTIC FLATNESS

NOTE: FOR OUTGOING SOLUTION, INSPIRAL SLOW ENOUGH THAT, OUT TO EDGE OF GRID, $E_{ENCLOSED}$ GROWS ~LINEARLY WITH DISTANCE AT APPROXIMATELY THE SAME RATE AS IN THE $\frac{1}{2}$OUTGOING + $\frac{1}{2}$INGOING SOLUTION.

AND MOST RADIATION IN PART OF SPACETIM[E] WHERE IT HAS LITTLE EFFECT ON ORBIT.

FOR THIS CONFIGURATION

HELICAL SYMMETRY $\Rightarrow$ NO ASYMPTOTIC FLATNESS

TIME-INDEPENDENT FLUX OF GRAVITATIONAL WAVES

$\Rightarrow$ M, J DIVERGE

WHAT DOES HELICAL MEAN IN CURVED SPACETIME?

NO NATURAL t^α OR ϕ^α FROM WHICH TO BUILD k^α. INSTEAD, USE

- k^α SPACELIKE OUTSIDE CYLINDER
- IF GO 'ONCE AROUND' COME BACK TO TIMELIKE SEPARATED POINT

DEF. A VECTOR FIELD k^α IS HELICAL IF THERE IS A SMALLEST $T > 0$ SUCH THAT P AND $\psi_T(P)$ ARE TIMELIKE SEPARATED, ALL $P \in M$

ψ_T FAMILY OF DIFFEOS GENERATED BY k^α

M SPACETIME MFLD

ALLOW BLACK HOLES, ERGOSPHERES WHERE $\psi_t(P)$ SPACELIKE SEPARATED FROM P:

CALL $\{\psi_t(S),\ t\in\mathbb{R}\}$ THE HISTORY OF $S\subset M$

DEF. A VECTOR FIELD k^α IS HELICAL IF THERE IS A SMALLEST $T>0$ SUCH THAT P AND $\psi_T(P)$ ARE TIMELIKE SEPARATED, ALL P OUTSIDE THE HISTORY OF SOME SPACELIKE SPHERE.

CONJECTURE $\exists$ EINSTEIN-EULER SPACETIMES WITH ONE KILLING VECTOR, WHICH IS HELICAL.

THESE INCLUDE JACOBI-LIKE CONFIGURATIONS AS WELL AS MODELS OF BINARY SYSTEMS OF 2 STARS, STAR + BLACK HOLE, 2 BHS.

OUR DEF OF A HELICAL VECTOR FIELD IS ESSENTIALLY EQUIVALENT TO SAYING THAT k^α IS HELICAL IF IT CAN BE WRITTEN IN THE FORM

$$k^\alpha = t^\alpha + \Omega \phi^\alpha ,$$

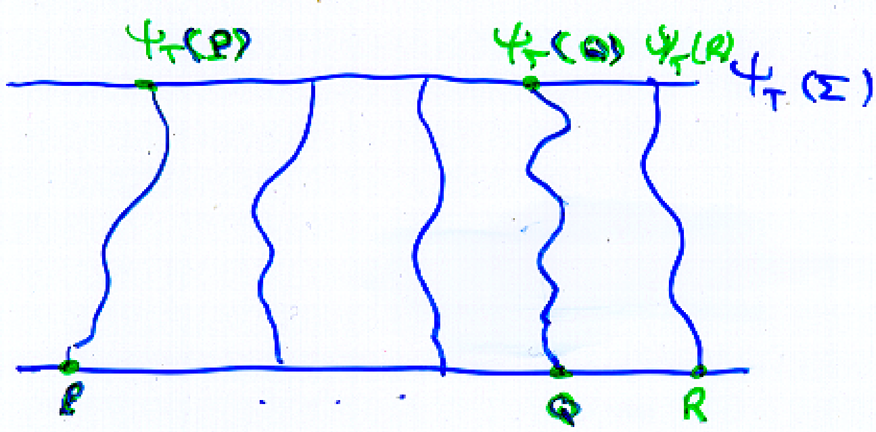
t^α TIMELIKE OUTSIDE THE HISTORY OF SOME SPHERE

ϕ^α ROTATIONAL (CIRCULAR ORBITS, $|k|$ UNBOUNDED)

BUT k^α CAN BE SO DECOMPOSED ONLY IF $\exists$ TIMELIKE CONGRUENCE JOINING P TO $\psi_T(P)$, ALL $P \in M \setminus$ HIST OF SPHERE

AND ONE CAN FIND COUNTEREXAMPLES.

t^α + ϕ^α ARE FAR FROM UNIQUE: e.g. EACH SMALL DIFFEO OF REGION BETWEEN HYPERSURFACE Σ + $\psi_T(\Sigma)$ THAT FIXES NBHDS OF Σ + $\psi_T(\Sigma)$ GIVES ANOTHER CHOICE.



TOY MODELS

1° DUST WITH INGOING FLUX = OUTGOING FLUX

$$T^{\alpha\beta} = \tfrac{1}{2}\rho\,(\ell^\alpha \ell^\beta + n^\alpha n^\beta)$$

AS EXPECTED, M GROWS LINEARLY IN SCHWARZSCHILD COORD r

$$ds^2 = -\left(\frac{r}{a}\right)^\alpha dt^2 + (1-e)^{-1}dr^2 + r^2 d\Omega^2,$$

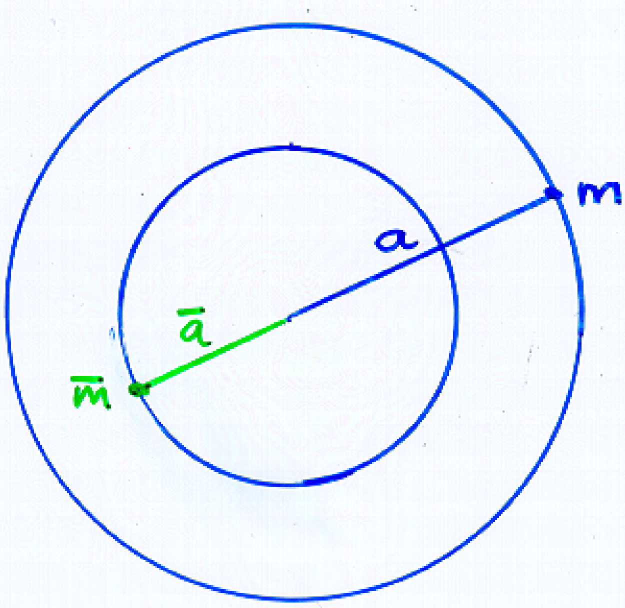
$$e = \frac{2M}{r} = \text{CONST}$$

2° BINARY IN MINKOWSKI SPACE W/ 2 POINT CHARGES
" " POST-MINKOWSKI APPROX W/ 2 POINT MASSES

RADIATION-REACTION PART OF SELF FORCE IS PART THAT CHANGES SIGN UNDER
ADVANCED ⟷ RETARDED
INGOING ⟷ OUTGOING

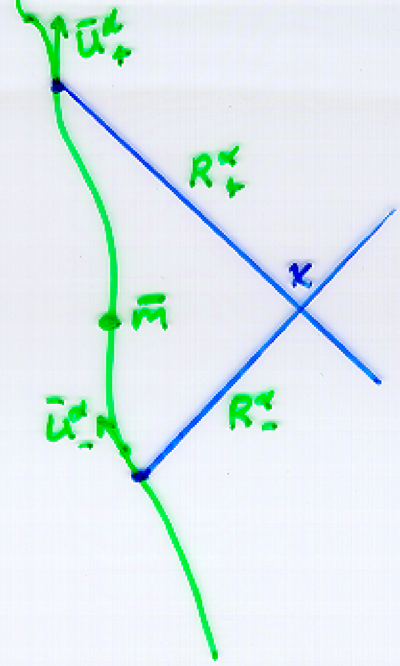
e-m SCHILD '63

GR URYU, TSOKAROS, JF '02



REMAINING SELF-FORCE HERE IS JUST MASS RENORMALIZATION

THEN FORCE ON m FROM $\left(\tfrac{1}{2}\text{ ADVANCED} + \tfrac{1}{2}\text{RETARDED}\right)\bar{h}_{\alpha\beta}$



$$h_{\alpha\beta} - \tfrac{1}{2}\eta_{\alpha\beta} h(x) = -4\bar m \frac{\bar u^\alpha \bar u^\beta}{\bar u_\gamma R^\gamma}$$

HERE u^α, $\bar u^\alpha$ ALONG k^α, THE HELICAL KV

$$u \cdot \nabla u = 0 \Rightarrow \nabla_\alpha [(g_{\beta\gamma} + \bar h_{\beta\gamma}) k^\beta k^\gamma] = 0$$

$$\Rightarrow \Omega = \Omega(a)$$

AGAIN RADIATION ENERGY DENSITY $\propto \frac{1}{r^2}$

ENCLOSED ENERGY AT $r \propto r$

CAVEATS

GR: EXPECT BINARY BH SOL'NS WITH HELICAL KV,

BUT: BINARY BLACK HOLES MUST BE COROTATING TO HAVE NO SHEAR AT HORIZON: IN BH SPACETIME W/ ONE KILLING VECTOR, THE KV COINCIDES ON EACH COMPONENT OF THE HORIZON W/ HORIZON'S GENERATORS.

i.e., OBSERVERS JUST OUTSIDE HORIZON, $u^\alpha \propto k^\alpha$, SEE STATIONARY METRIC, NO RADIATION.

AND NO WAY TO PICK A UNIQUE MEMBER OF SOL'NS WITH HELICAL KV, ANALOGOUS TO $\frac{1}{2}$ ADVANCED + $\frac{1}{2}$ RETARDE