“Many black hole” space-times

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based on work in collaboration with
Erwann Delay and Rafe Mazzeo

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Brill-Lindquist initial data: the space-metric at time $t = 0$ takes the form

$$g = \psi^4(dx^2 + dy^2 + dz^2) ,$$

with

$$\psi = 1 + \sum_{i=1}^{I} \frac{m_i}{2|\vec{x} - \vec{x}_i|} .$$

A special case: the space-part of the Schwarzschild metric centred at $\vec{x}_0$ with mass m :

$$g = \left(1 + \frac{m}{2|\vec{x} - \vec{x}_0|}\right)^4 \delta ,$$

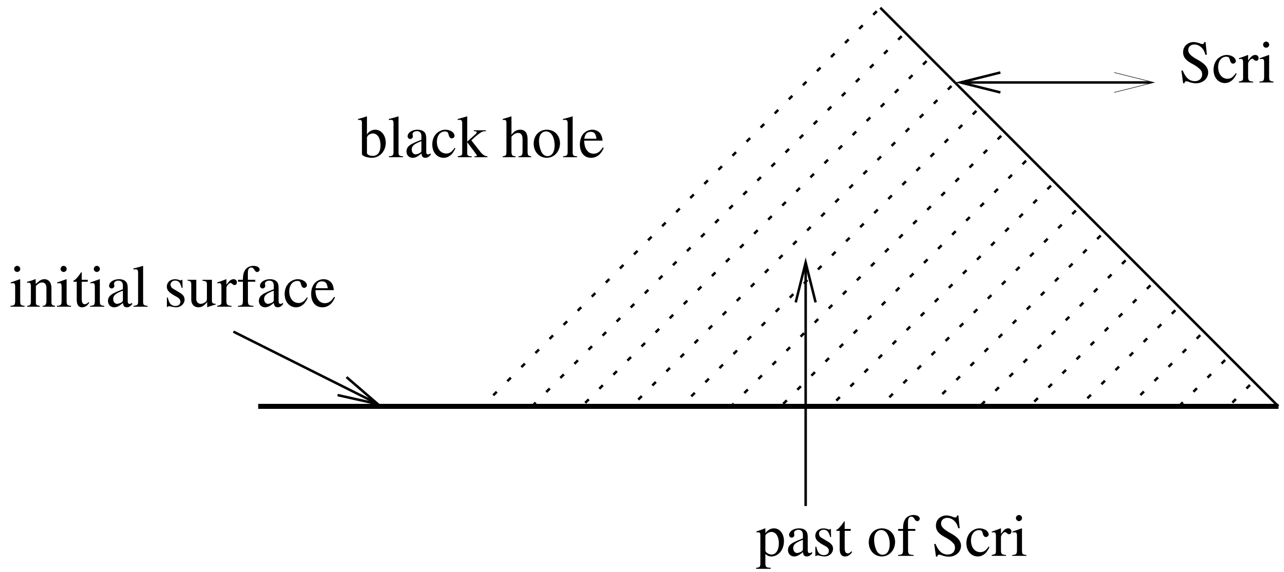
where δ is the Euclidean metric.

Question: How differentiable is the associated Scri?

What is a black hole in a space-time $(\mathscr{M}, g)$*?*

$\mathscr{B}$ — black hole region

$\mathscr{B} := \mathscr{M} \setminus J^-(\mathscr{I}^+)$



$\mathscr{I}^+$ – future null infinity Scri,

cf. Friedrich's and Galloway's lectures

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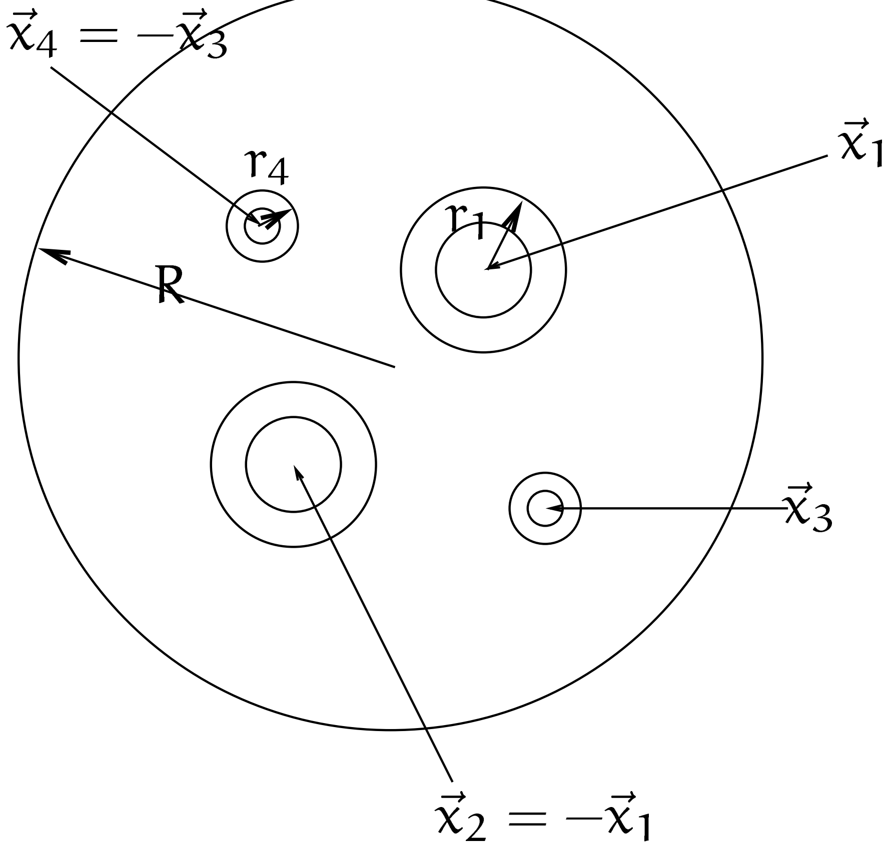
The "many Schwarzschild metrics" (PTC, E.Delay, CQG, gr-qc/0203053) a solution exists when $0 \le m_i \le \delta$, δ small enough



Free parameters: R and $(\vec{x}_1, r_1, m_1)$, $(\vec{x}_3, r_3, m_3)$

$m_2 = m_1$, $r_2 = r_1$
$m_4 = m_3$, $r_4 = r_3$

inner small circles are Schwarzschild horizons; metric *exactly Schwarzschild* within the outer small circles and outside the big circle

Recall:

- for time-symmetric initial data ($K_{ij} = 0$):

  apparent horizon = outermost minimal surface

- **If** there exists a sufficiently regular conformal completion Scri, then

  apparent horizon $\qquad \Longrightarrow \qquad$ black hole region

Theorem: (PTC, R.Mazzeo, in preparation) If the m_i's $i =, 1 \ldots I$, are small enough, then the *apparent horizon* (= outermost minimal surface) has at least (precisely?) I connected components.

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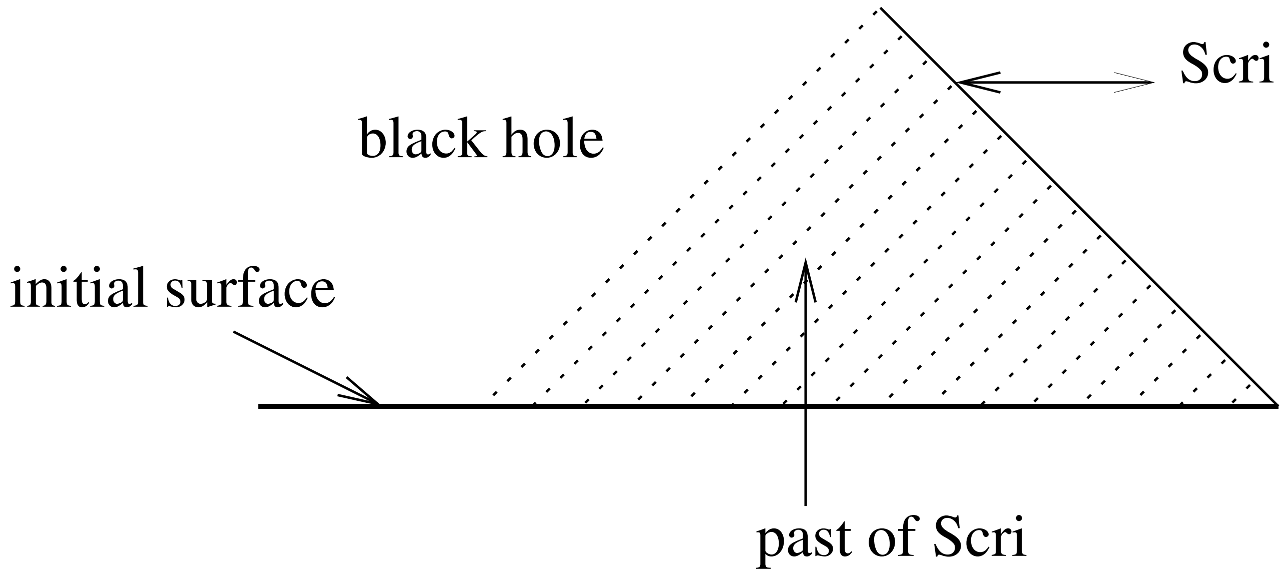

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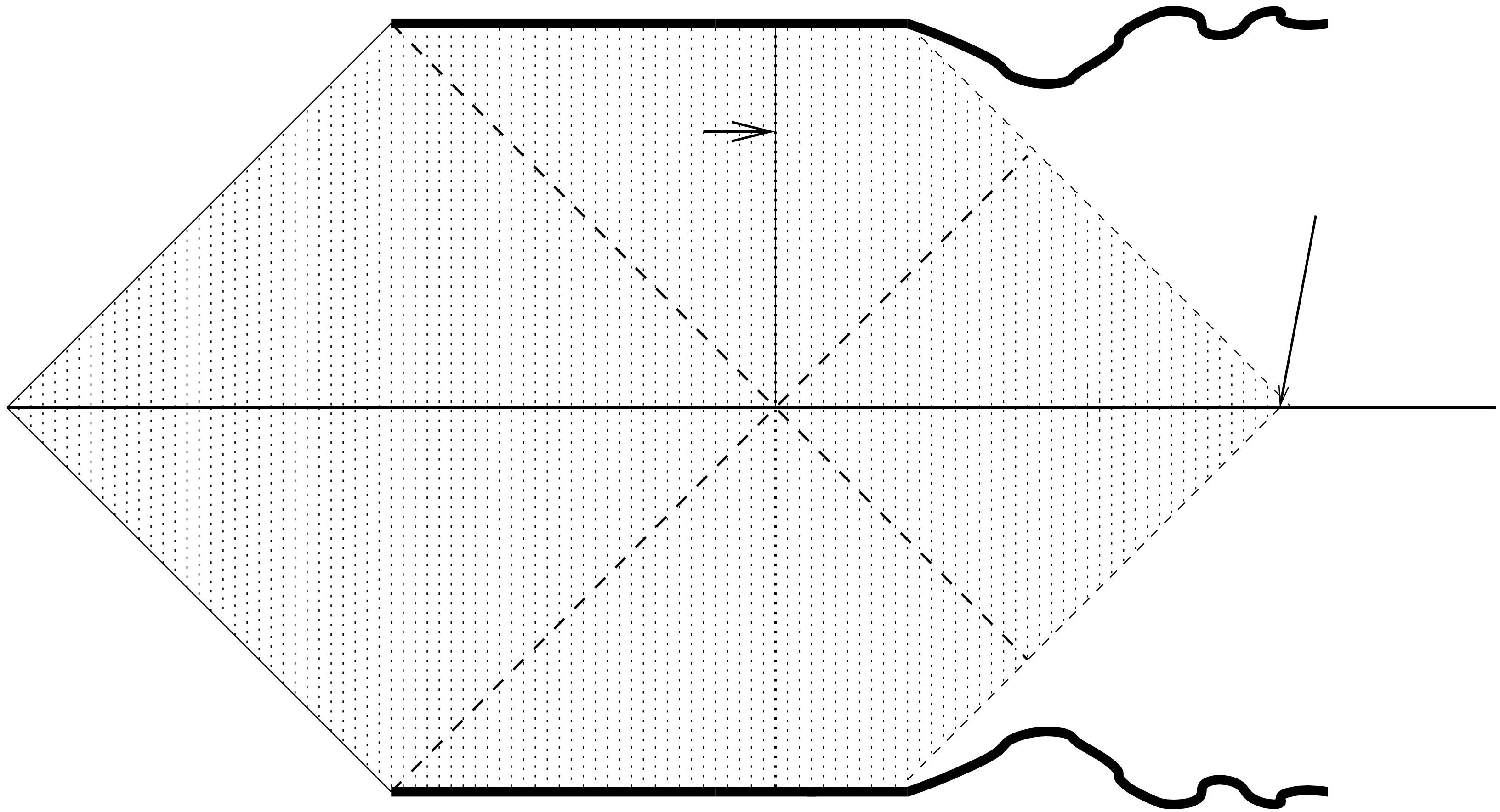
What about event horizons? The resulting space-times are "small":

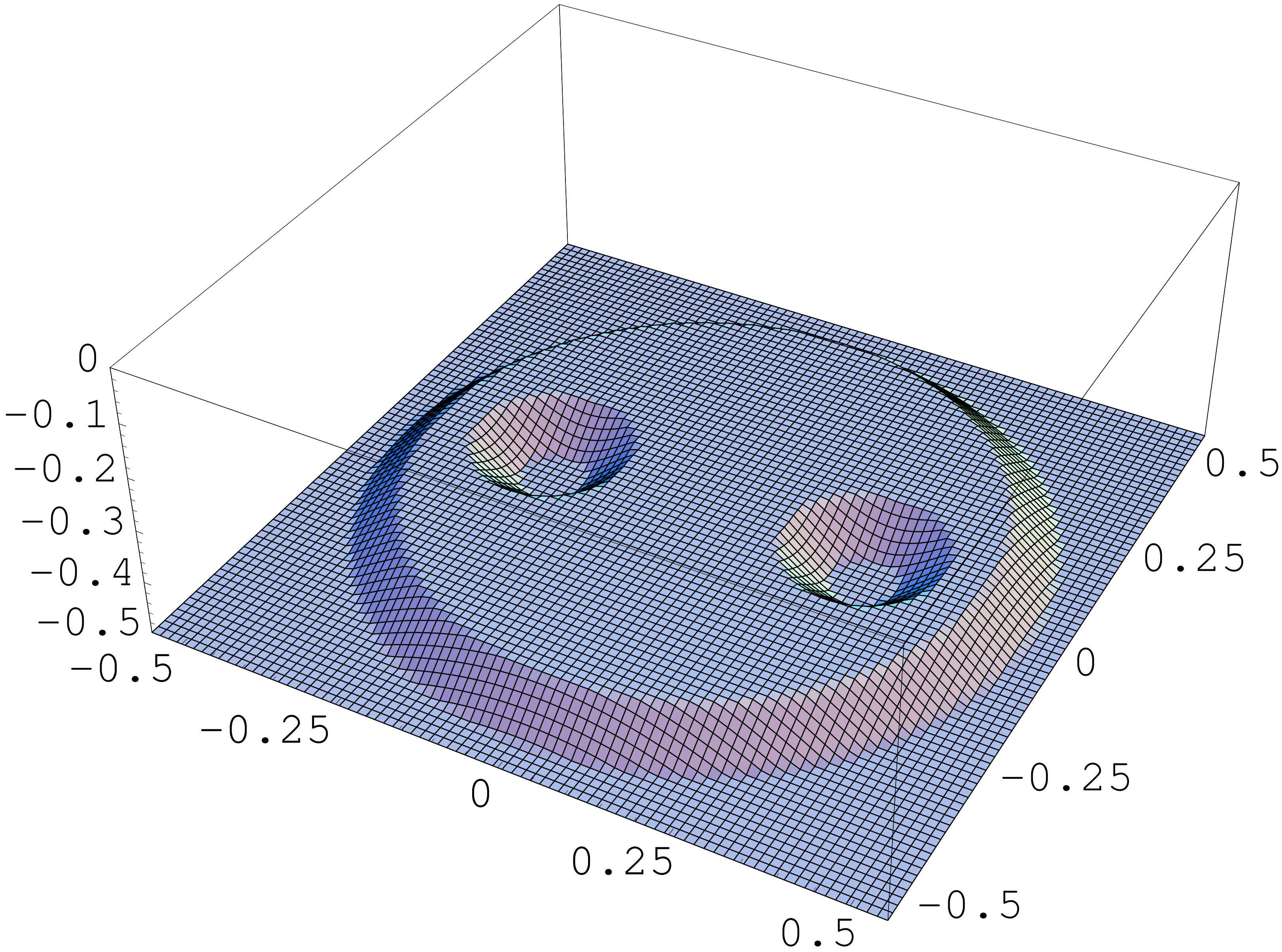


Theorem (PTC, R.Mazzeo, in preparation)

There exist configurations with *at least* I connected components of the intersection of the event horizon with the initial data hypersurface:

- two black holes, m_i small enough
- all black holes aligned, m_i small enough.

Why is there a problem? A possible global structure:



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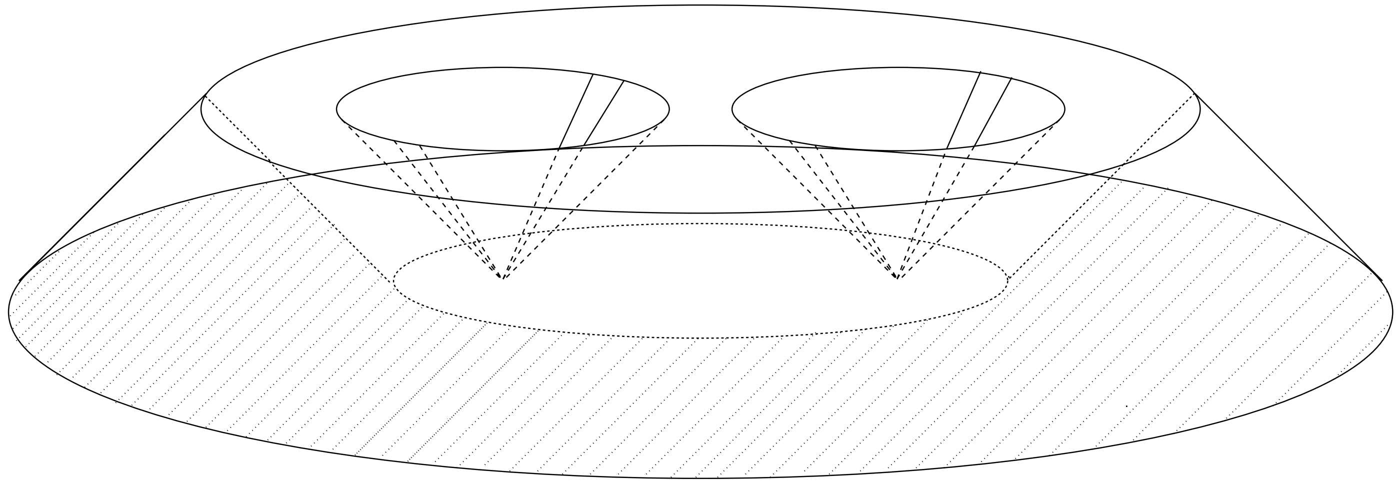

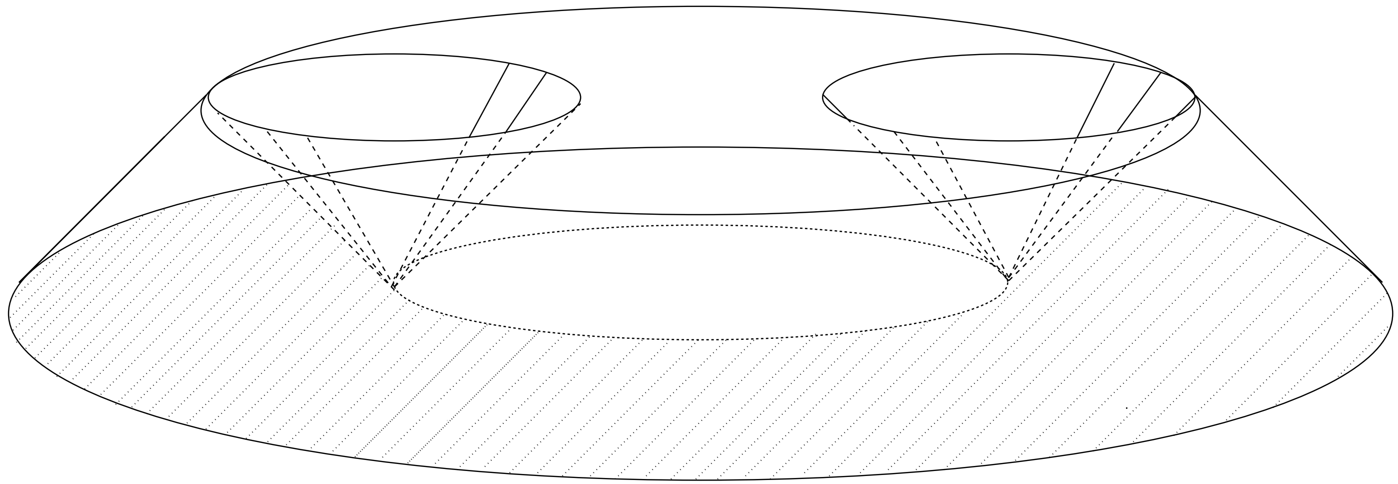

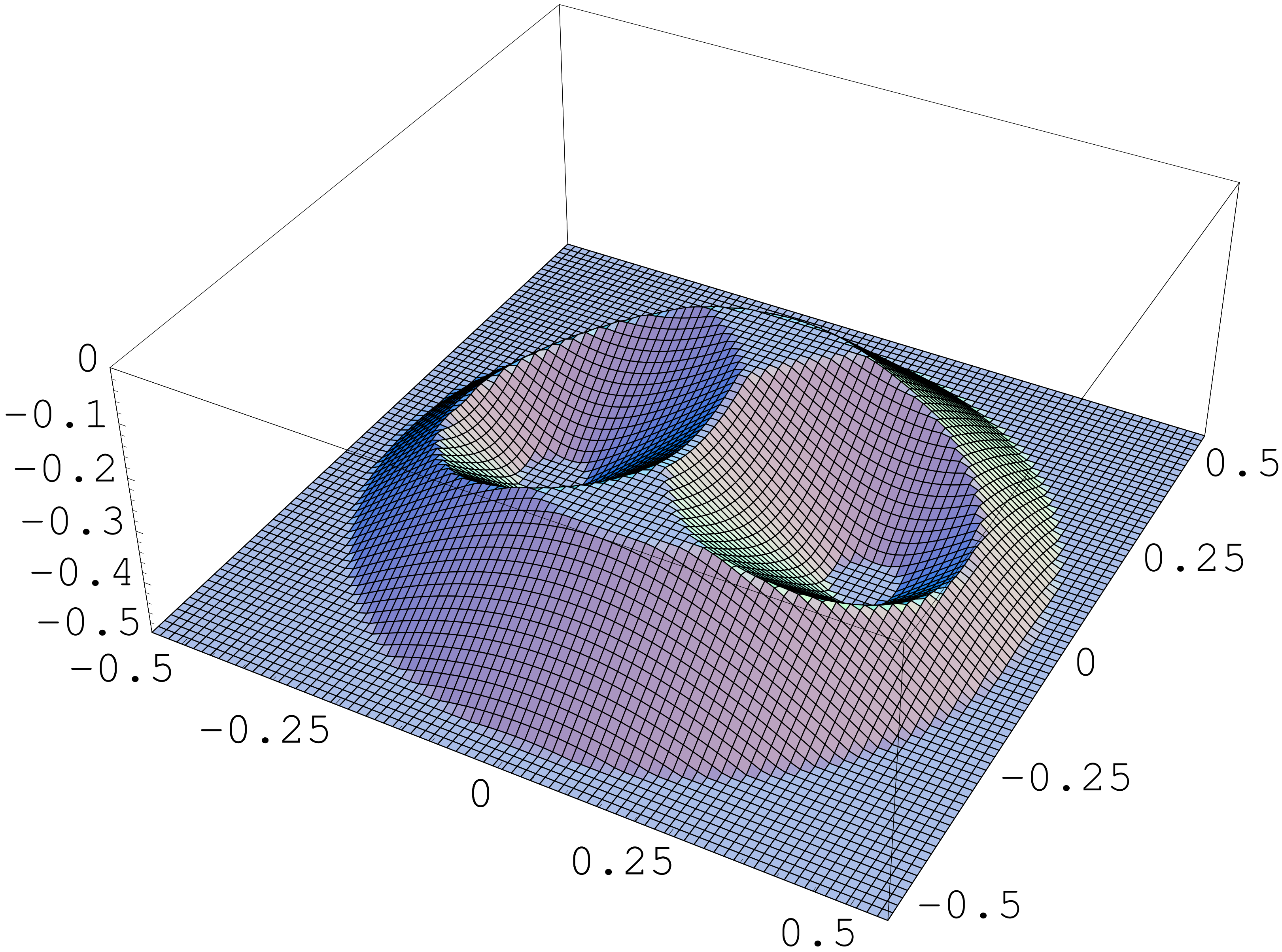
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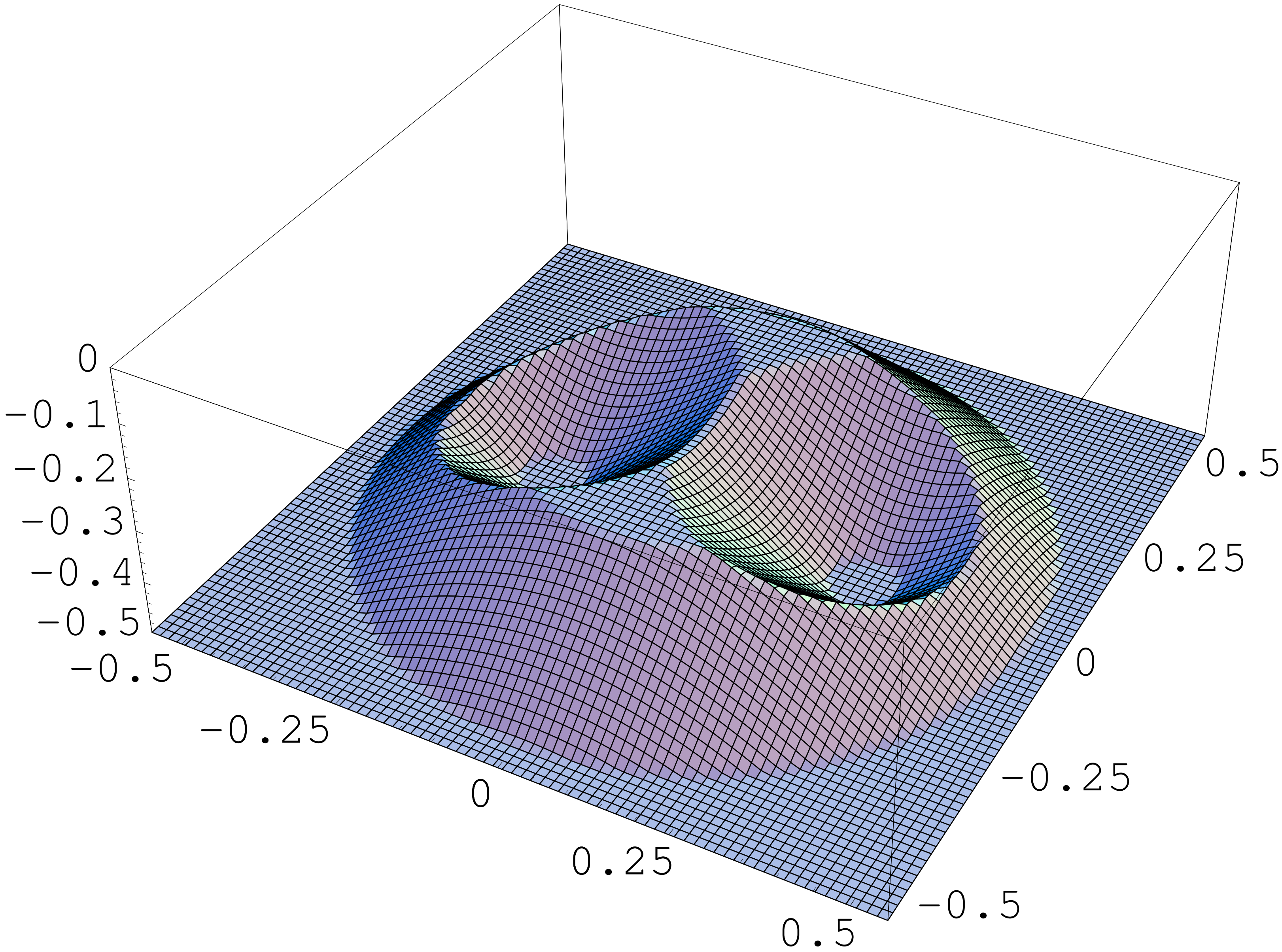
A simple way of obtaining (a piece of) the Minkowskian Scri:

$$\{x^0 > 0 \, , \, \eta_{\alpha\beta} x^\alpha x^\beta < 0\} \ni x^\mu \to y^\mu = \frac{x^\mu}{\eta_{\alpha\beta} x^\alpha x^\beta} \in \{y^0 < 0 \, , \, \eta_{\alpha\beta} y^\alpha y^\beta < 0\} \, .$$

Here $\eta_{\mu\nu}$ is the Minkowski metric.

$y^\mu = 0$ is a point representing the "timelike future infinity", usually denoted by i^+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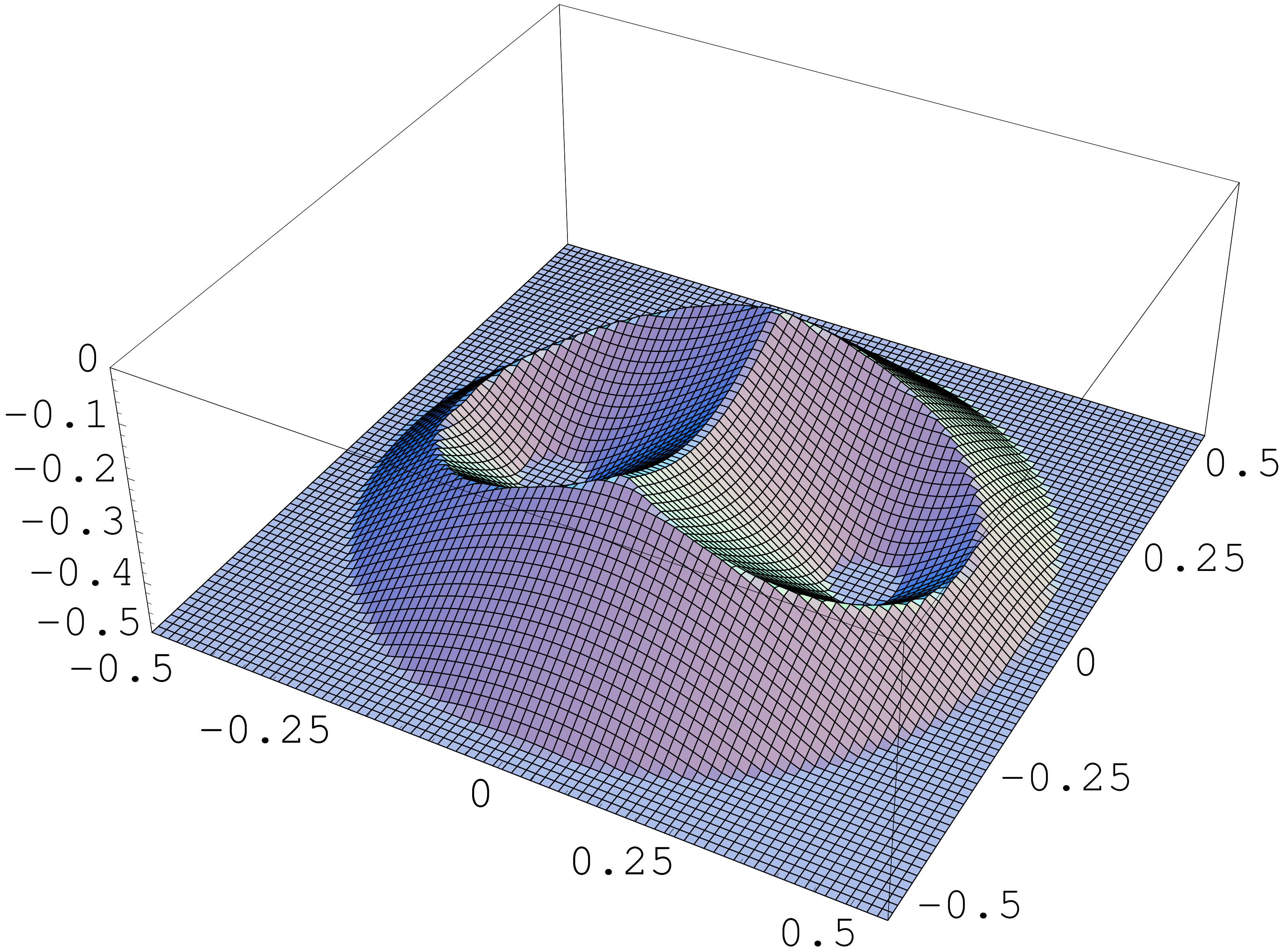
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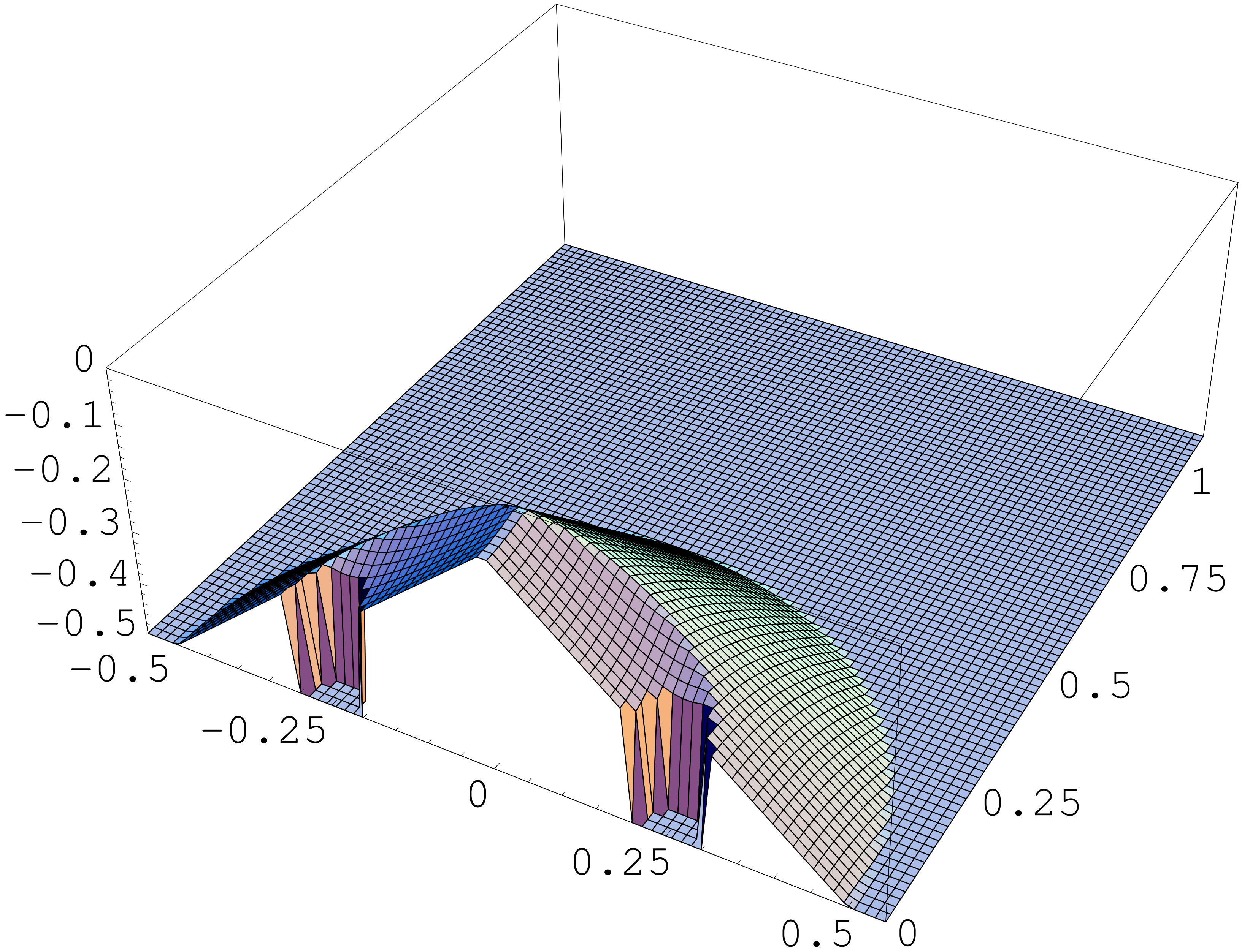
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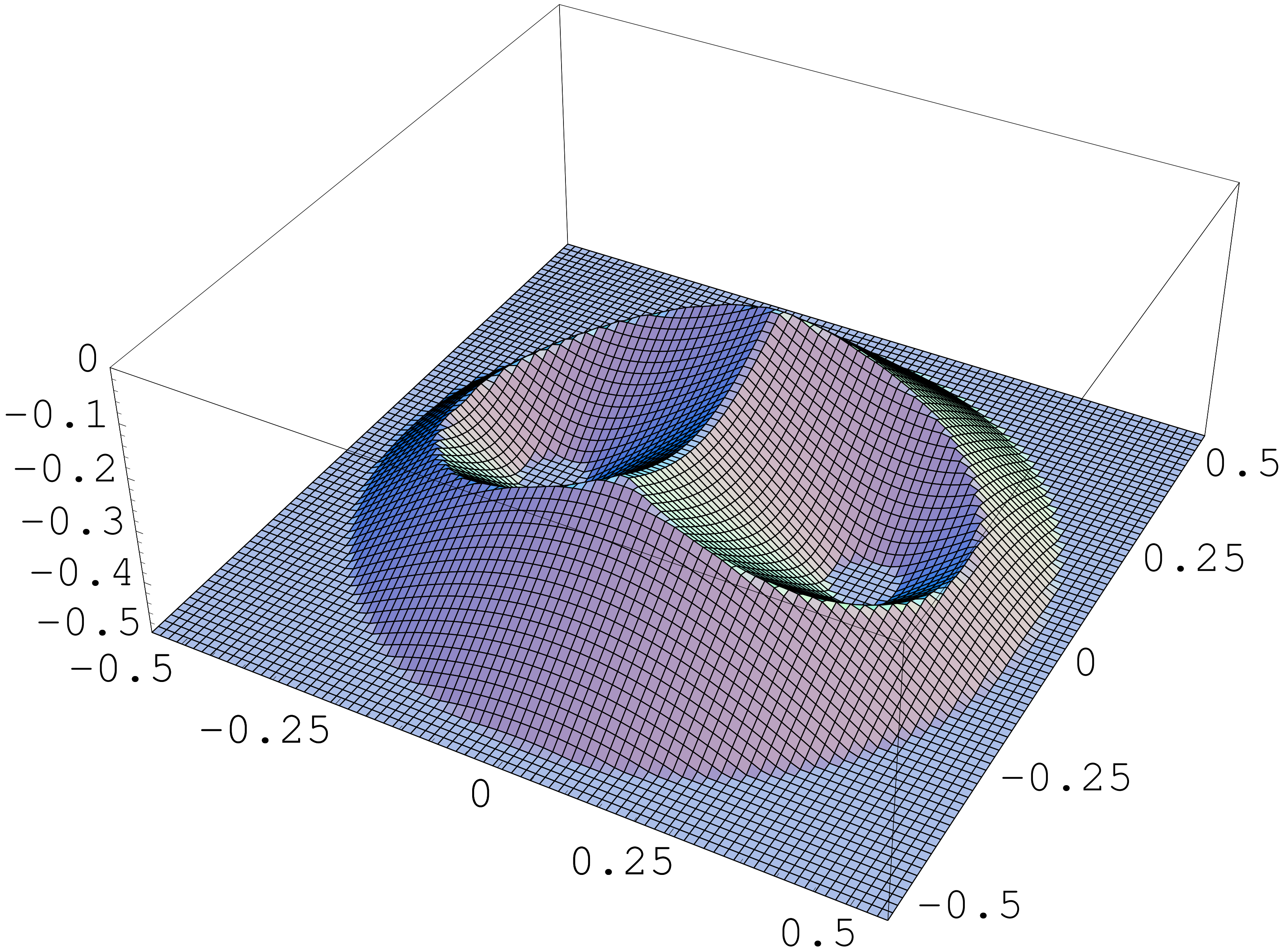
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