

Light cones in cosmology

(Gravitational optics)

Cargèse Summer School 2002

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Selected References on  
Foundations of Gravitational Lens Theory

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~~Stewart, J. M.~~

# The role of light cones in <sup>cosmology</sup> ~~geometry~~

(gravitational optics)

Purpose: i) Show how Loventrian geometry is used to express in mathematically idealized form the ~~main~~ relations between measurables and (astrophysical) theoreticals, especially in cosmology.

ii) Show how gravitational lens phenomena arise ~~the~~ within GR from the fact that light cones frequently have caustics and self intersections

This will be done qualitatively, mostly in terms of spacetime diagrams which convey information more easily than equations.

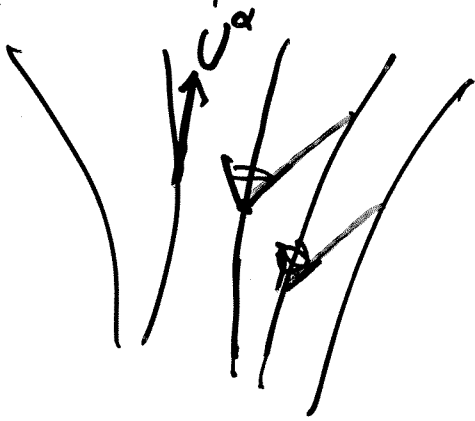
My reason to present this material, ~~is~~ which ought to be graduate text book stuff, is that, due to tradition, the relations in question are presented <sup>without any need</sup> by using from the start "approximations" based on plausibility arguments. I feel that it is preferable to describe these relations without approximations until one is forced to use to introduce them. ~~I shall mention them later.~~ This has the advantage that one better understands what

one is talking about and what one wants  
to approximate. I shall mention the usual errors  
briefly ~~later~~.  
To emphasize that what I am talking  
about <sup>concerns</sup> the cosmology I will show some  
beautiful pictures of lens phenomena.

# Gravitational Optics

The setting:

$(M, g_{\alpha\beta}, U^\alpha) =$  Kinematic cosmological model



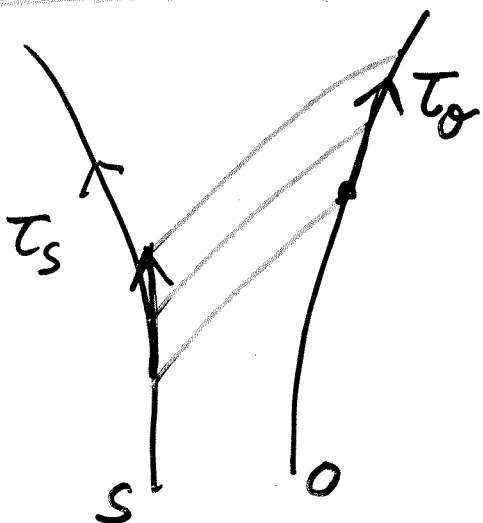
Mean motion of sources and observer  
→ light rays

## Geometrical Optics

(from Maxwell's eqs.)

- i) light rays  $\leftrightarrow$  null geodesics
- ii) (Classical) photons carry 4-momentum  

$$P^\alpha = \frac{dx^\alpha}{d\tau}$$
;  $E = -P \cdot U$   
 $\perp$  observer
- iii) photons conserved



$$1+z := \frac{d\tau_o}{d\tau_s} = \frac{(P \cdot U)_s}{(P \cdot U)_o} = \frac{\lambda_o}{\lambda_s}$$

$\tau_o, \lambda_o$  measurable

$\lambda_s$  theoretically inferred

(sometimes  $\Delta\tau_s$  " )

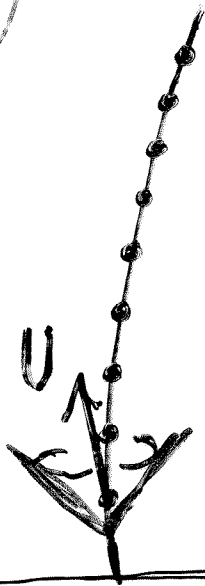
Anerkennung machte ihn zum Mitherausgeber mehrerer angesehenen wissenschaftlicher Journale, zum Chairman oder Mitglied der Programm Komitees internationaler Konferenzen, zum Fachausschuß-Vorsitzenden „Halbleiterphysik“ und zum Sprecher des Arbeitskreises „Festkörperphysik“ in der Deutschen Physikalischen Gesellschaft.

Der junge Hochschullehrer war nicht nur ein produktiver Forscher, sondern er entfaltete Anfang der siebziger Jahre ein ideenreiches Programm zur Reform des Physikstudiums. Die enge Verflechtung von experimenteller und theoretischer Ausbildung wurde schnell zu einer Besonderheit, ja Berühmtheit des Dortmunder Physikfachbereichs. Die Studierenden nahmen sie begeistert auf. Für zahlreiche spätere Reformbestrebungen wurde sie erfolgreiches Vorbild. Dabei zeigte sich auch, wie sehr diese Reform von der Ausstrahlung der Person Treusch getragen worden war.

Der mitreißende Lehrer und Forscher hatte seinen Platz unter den Physikern gefunden. Mit 44 Jahren wählten sie ihn zum Präsidenten der Deutschen Physikalischen Gesellschaft. Und auch hier entfaltete er Treuschsches ideenreiches Engagement: Durch besonderes Ansprechen der jungen Physiker blühte die DPG rasch auf, verjüngte sich, gewann Attraktivität und Sichtbarkeit, machte sich auf den Weg zur derzeit zweitgrößten Physikalischen Gesellschaft der Welt.

1987 erfuhr die eindrucksvolle Karriere von Joachim Treusch als erfolgreicher Forscher und Lehrer eine ganz neue Ausrichtung. Als er in den Vorstand der damaligen KFA Jülich berufen wurde, endete zwar bald die Liste seiner fachspezifischen Publikationen. Seine gestalterische Kraft widmete er aber nun der strategischen Entwicklung der Forschungslandschaft, insbesondere der Forschungsperspektiven in einem der größten Zentren. Er tat dies ab 1990 als Vorsitzender des Vorstands des Forschungszentrums Jülich. Seine Zuständigkeiten entsprechen seiner Interessenvielfalt und Breite: Physikalische Grundlagenforschung – klar, aber auch Informationstechnologie und neuen Wege hierin, ferner Umwelt, ferner Lebenswissenschaften. Seinem Zentrum gab er mit neuen Zielen und zukunftsweisenden Aufgaben neuen Optimismus. 1993 führte seine Überzeugungskraft den eher lockeren Zusammenschluß der Arbeitsgemeinschaft der deutschen Großforschungszentren, AGF, über in die Helmholtzgemeinschaft der deutschen Forschungszentren, HGF. Er gestaltete als ihr erster Vorsitzender eine neue Qualität der Zentrenvernetzung, eine neue Sichtbarkeit und einen Umschwung in der öffentlichen Wahrnehmung sowie der politischen Anerkennung der großen Forschungszentren. Derzeit befindet sich die Helmholtzgemeinschaft in einer interessanten,

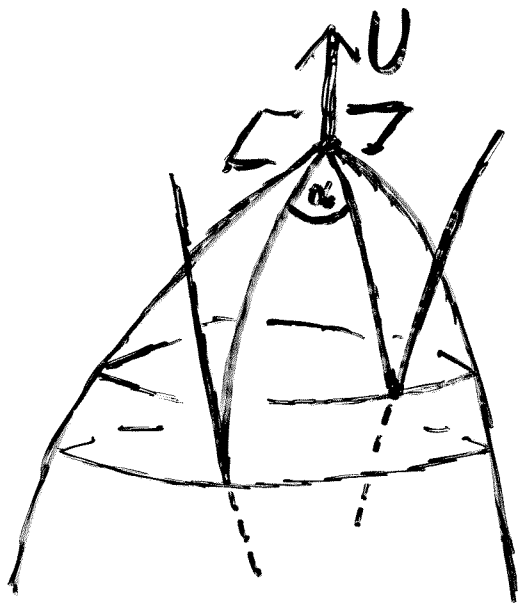
②



Observer, world line  
(timelike), 4-velocity;  
Clock, proper time:

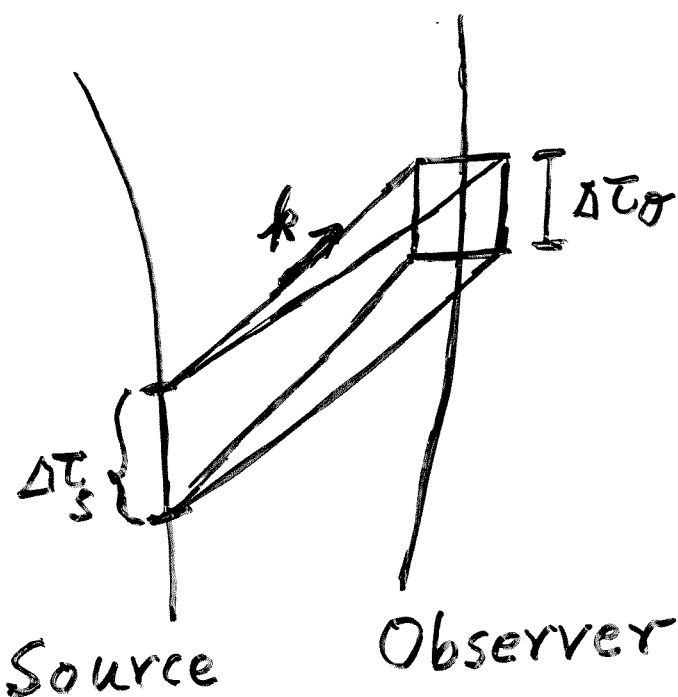
$$\tau = \int |g_{\alpha\beta} dx^\alpha dx^\beta|^{\frac{1}{2}}$$

Chronometry



Angle between  
light rays

$$\sin\left(\frac{\alpha}{2}\right) = \left\{ \frac{k_1 \cdot k_2}{2(U \cdot k_1)(U \cdot k_2)} \right\}^{\frac{1}{2}}$$



Frequency ratio

$$\frac{\nu_o}{\nu_s} = \frac{d\tau_s}{d\tau_o} = \frac{(k \cdot U)_o}{(k \cdot U)_s}$$

Energy received

$$E = \int T^{\alpha\beta} U_\beta da_\alpha$$

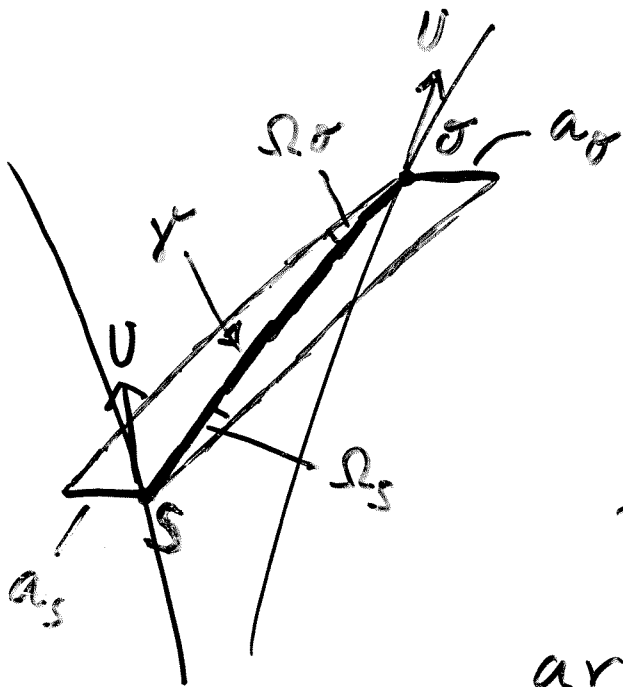
area × time } at observer



③

# Distances in

## curved spacetime / astronomy



Affine distances,

$$r' := \left( \frac{a_U}{\Omega_U} \right)^{\frac{1}{2}},$$

corrected luminosity distance;  $r'(\sigma, S, U_S)$

$$r := \left( \frac{a_S}{\Omega_S} \right)^{\frac{1}{2}}, \quad r(\sigma, S, U_S)$$

area distance (angular distance)

Thm. (Etherington 1933):

$$r' = (1+z)r$$

$r = r(z)$ , depends on  $g_{\alpha\beta}$  near ray  $\gamma$

Tool: Geodesic deviation equ.

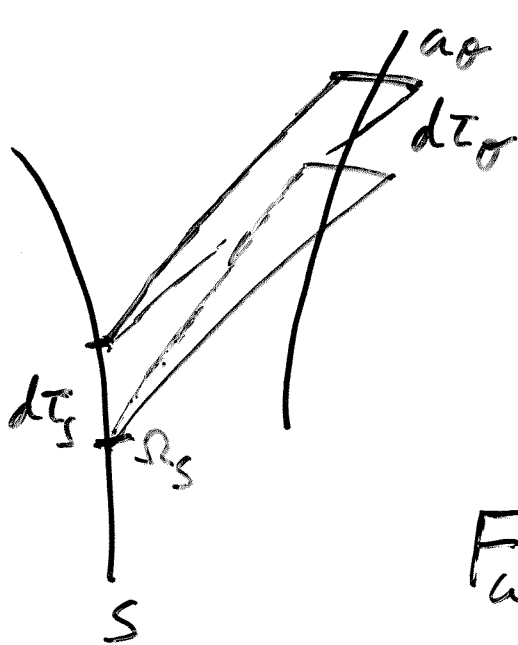
$$\nabla_s^2 Y^\alpha = R^\alpha_{\beta\gamma\delta} k^\beta k^\gamma Y^\delta$$

$$\Rightarrow \int \nabla_s^2 r = -(|\sigma|^2 + \frac{1}{2} R_{\alpha\beta} k^\alpha k^\beta) r$$

$$\left\{ \begin{array}{l} \dot{\sigma} + k^\alpha_{; \alpha} \sigma = C_{\text{curv}} \text{ (conf. curv.)} \end{array} \right.$$

$\Rightarrow$  Conj. points

④

Photon conservation

$$d\tau_s \Omega_s d\omega_s \frac{L_{\omega,s}}{4\pi \omega_s}$$

$$= d\tau_r \Omega_r d\omega_r \frac{F_{\omega,r}}{\omega_r}$$

$$F_{\omega}(\theta) = \frac{L_{(1+z)\omega}(s)}{4\pi (1+z)^3 r^2}$$

$$\boxed{F(\theta) = \frac{L(s)}{4\pi (1+z)^4 r^2}} \quad (\text{bolometric})$$

$$\Omega_r = \frac{a_s}{r^2}, \quad \underline{I(\theta) = \frac{F}{\Omega} = \frac{I(s)}{(1+z)^4}}$$

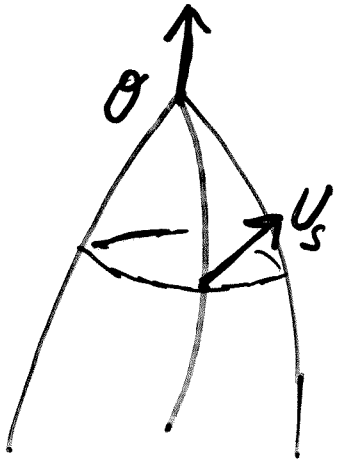
Needed both  $r'$  and  $r$  and  
Ether. thm.

Again: gap enters  $z \leftrightarrow r$ .

Measurables: time, angle, energy.

Theoreticals:  $a_s, \omega_s, L, I(s)$ .

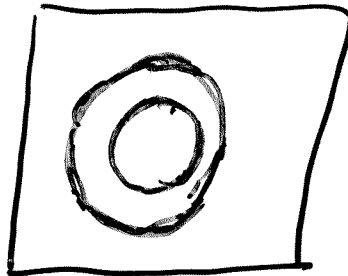
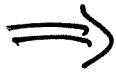
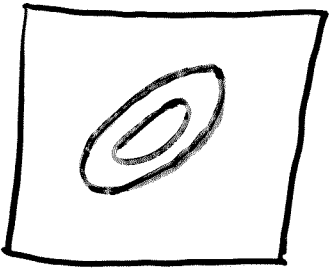
⑤ In a kinematical cosmological model, "in principle" determined:



$$r = r(\vec{e}, z)$$

$$(\nu(s), z(s), s(z) \text{ for } \vec{e})$$

$$L = F \cdot f(\vec{e}, z)$$



Isophotes  
measured  
 $I(\theta) = \text{const.}$

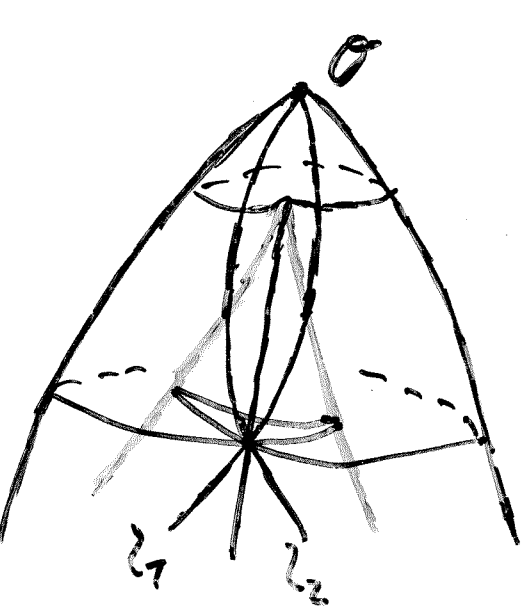
Isophotes  
from theory  
 $I(s) = \text{const.}$

Needed: null geodesics,  
 $R_{\alpha\beta}, C_{\alpha\beta\gamma\delta}.$

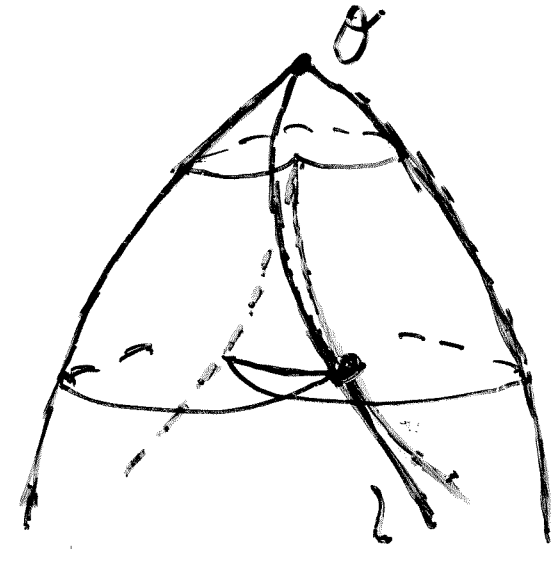
⑥ Theory: Geometrical optics  
in curved spacetime,  
geometry of light cone shapes

(Newman, Frittelli, Perlick, J.E)  
 ≤95

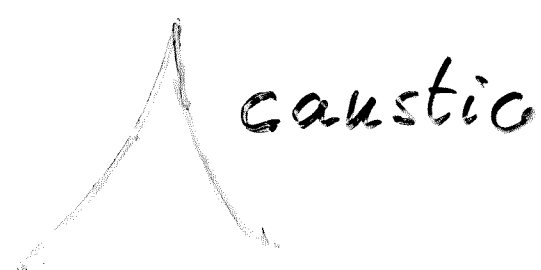
Singular points on a light  
 cone:



intersection  
point,  $l_1, l_2$



caustic point,  
conjugate to  $\sigma$   
on  $l$  ( $D=0$ )



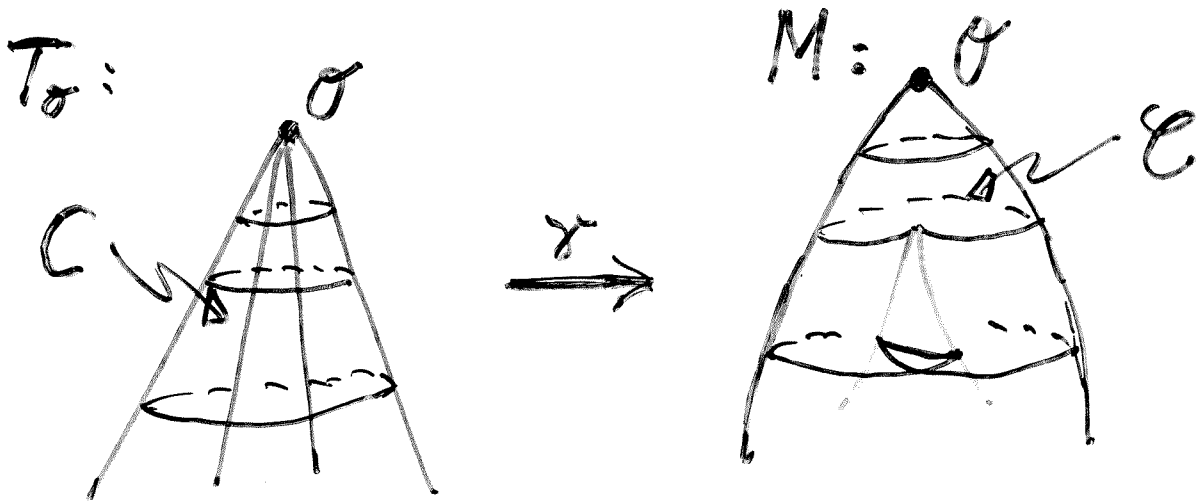
caustic

(2-dimensional  
 sheets)

The light "cone"  
in general is  
not smooth.

Geodesic deviation

⑦ The light cone  $\mathcal{E}$  in spacetime  $M$  as image of a null cone  $C$  in the tangent space  $T_\sigma$  of its apex  $\sigma$ .



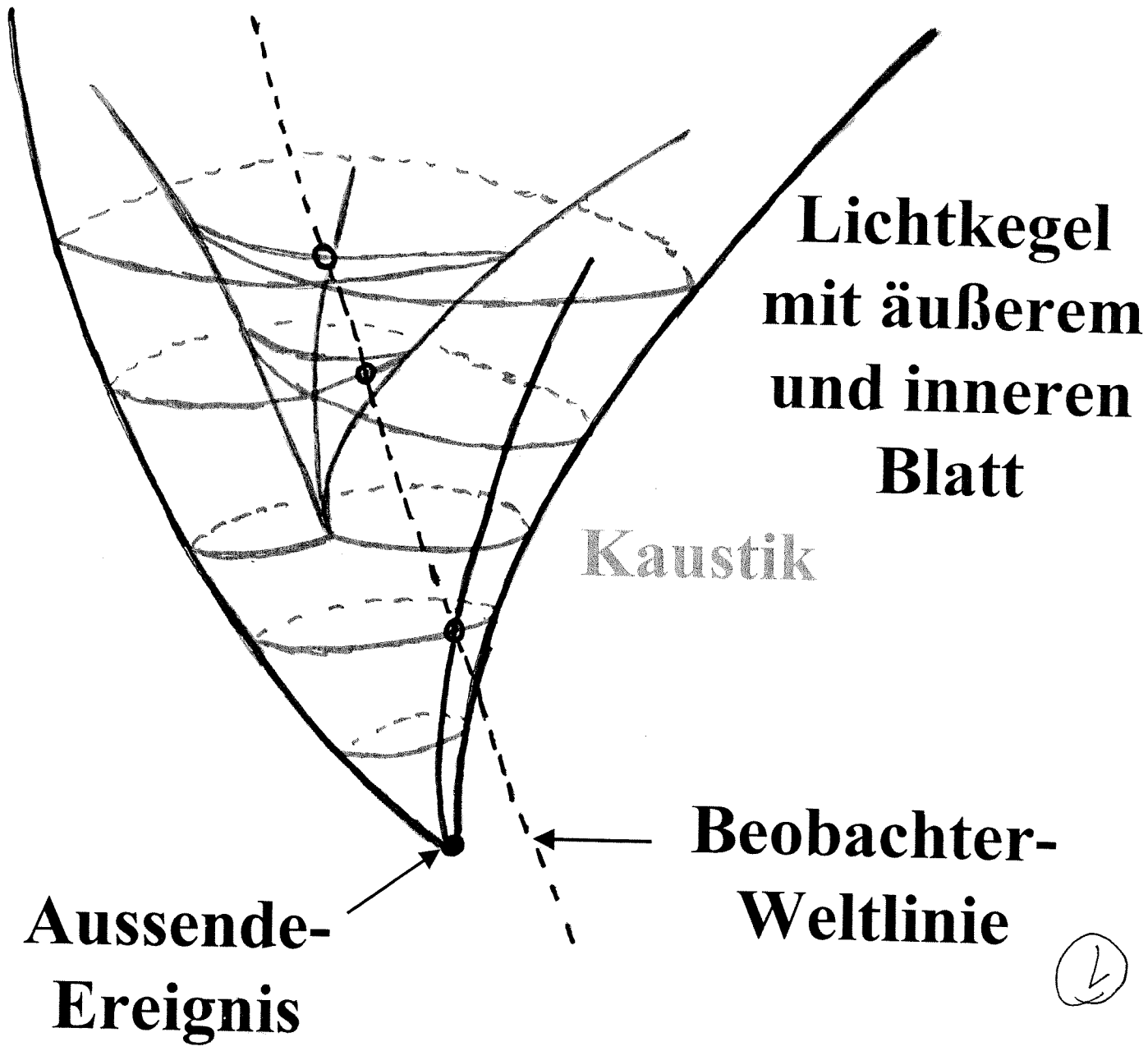
$$\gamma = \exp_{\sigma}|_C, \quad \mathcal{E} = \gamma(C).$$

$\gamma$  is a Legendre map;

$\Rightarrow$  5 stable singularities  
for  $\dim M = 4$ . (Arnold, 1985)

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# Dreimaliger Empfang eines Lichtblitzes durch einen Beobachter



**Aussende-Ereignis**

**Lichtkegel mit äußerem und innerem Blatt**

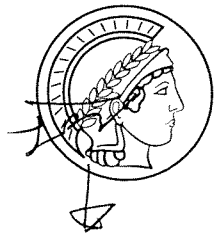
**Kaustik**

**Beobachter-Weltlinie**

①

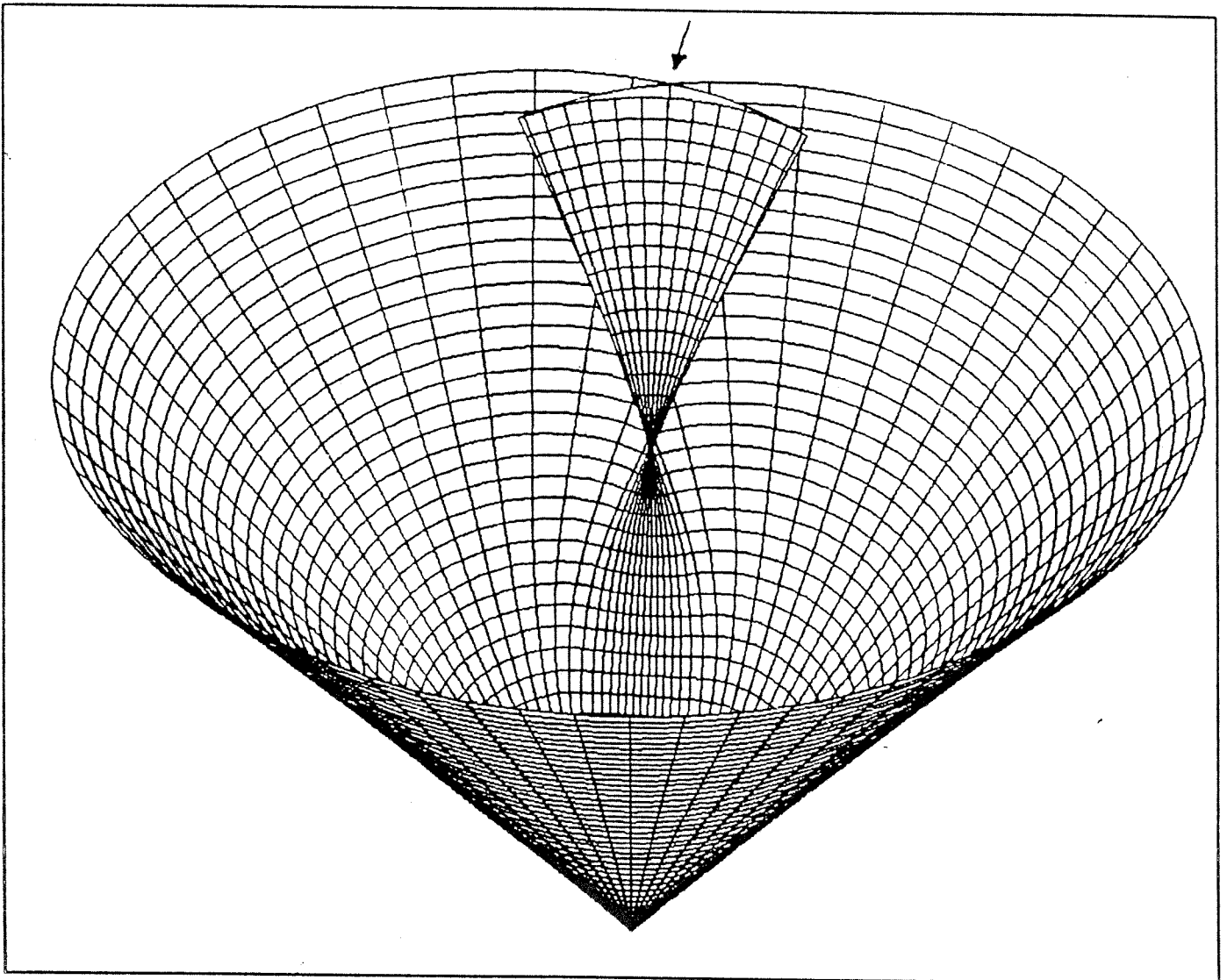


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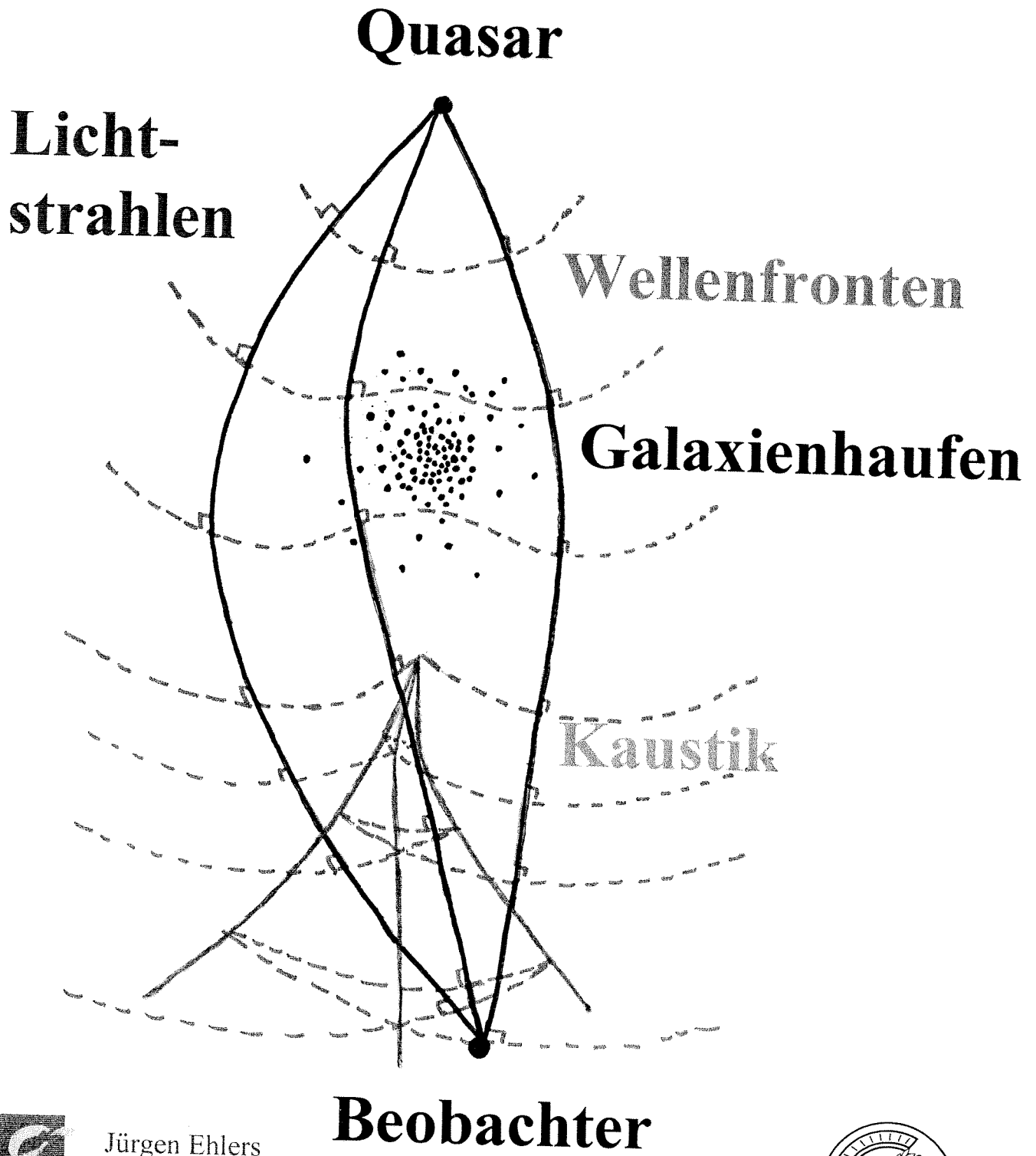


*von der*

⑨ Light cone in  $(1+2)d.$  spacetime



# Gravitative Lichtablenkung



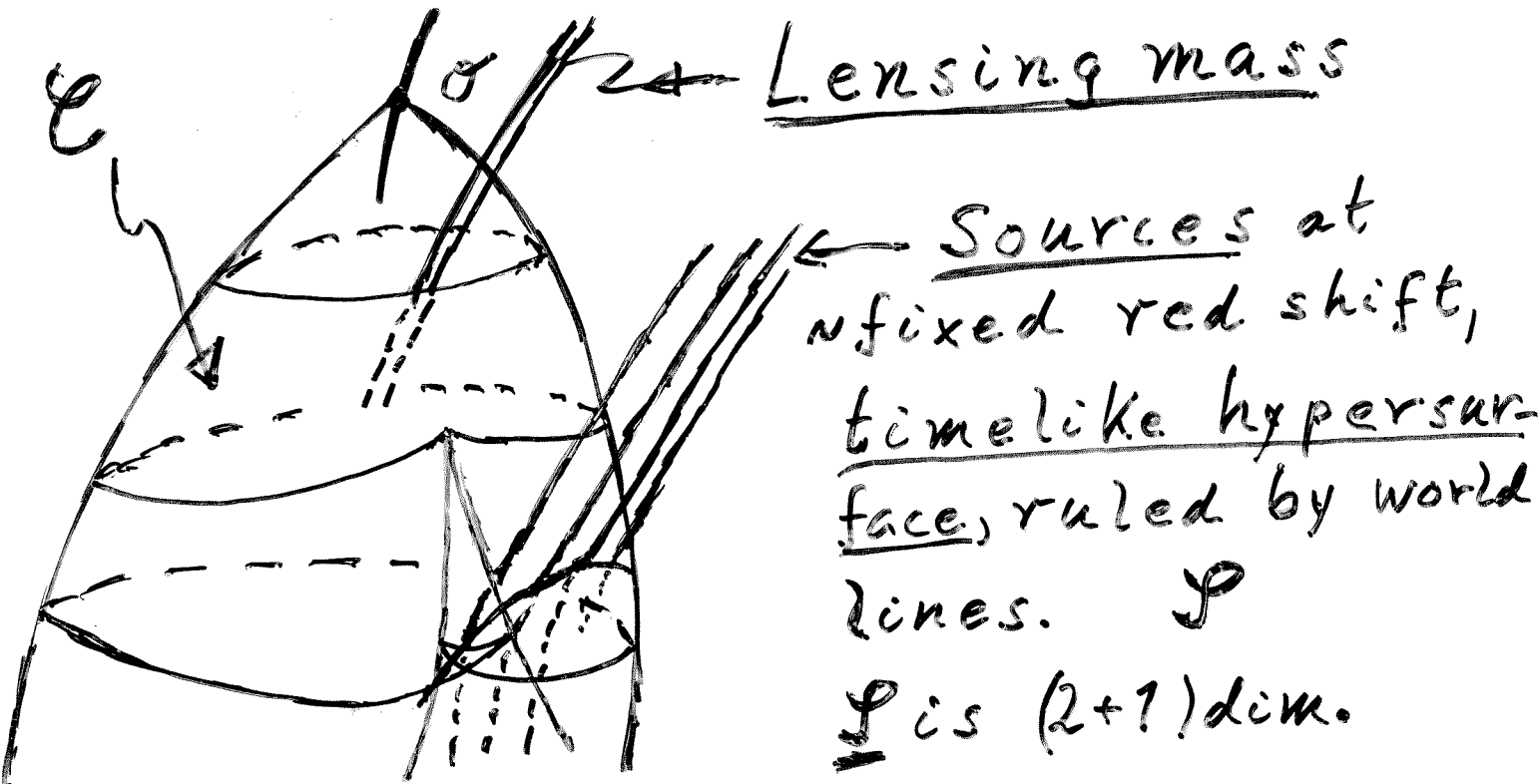
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(11)

# The lensing phenomenon



$\mathcal{E} \cap \mathcal{S}$  is the emission wave front  
 direction  $\vec{e}$  of light ray at  $\mathcal{O}$   
 $\leftrightarrow$  points of  $\mathcal{E} \cap \mathcal{S}$ .

$\Rightarrow \underline{\lambda} : S^2 \rightarrow \mathcal{S}, \lambda \text{ Leg. map}$

general lens map

$\vec{e} \mapsto (\tau, y) \in \mathcal{E} \cap \mathcal{S}$   
 $\uparrow \quad \uparrow$   
 time position

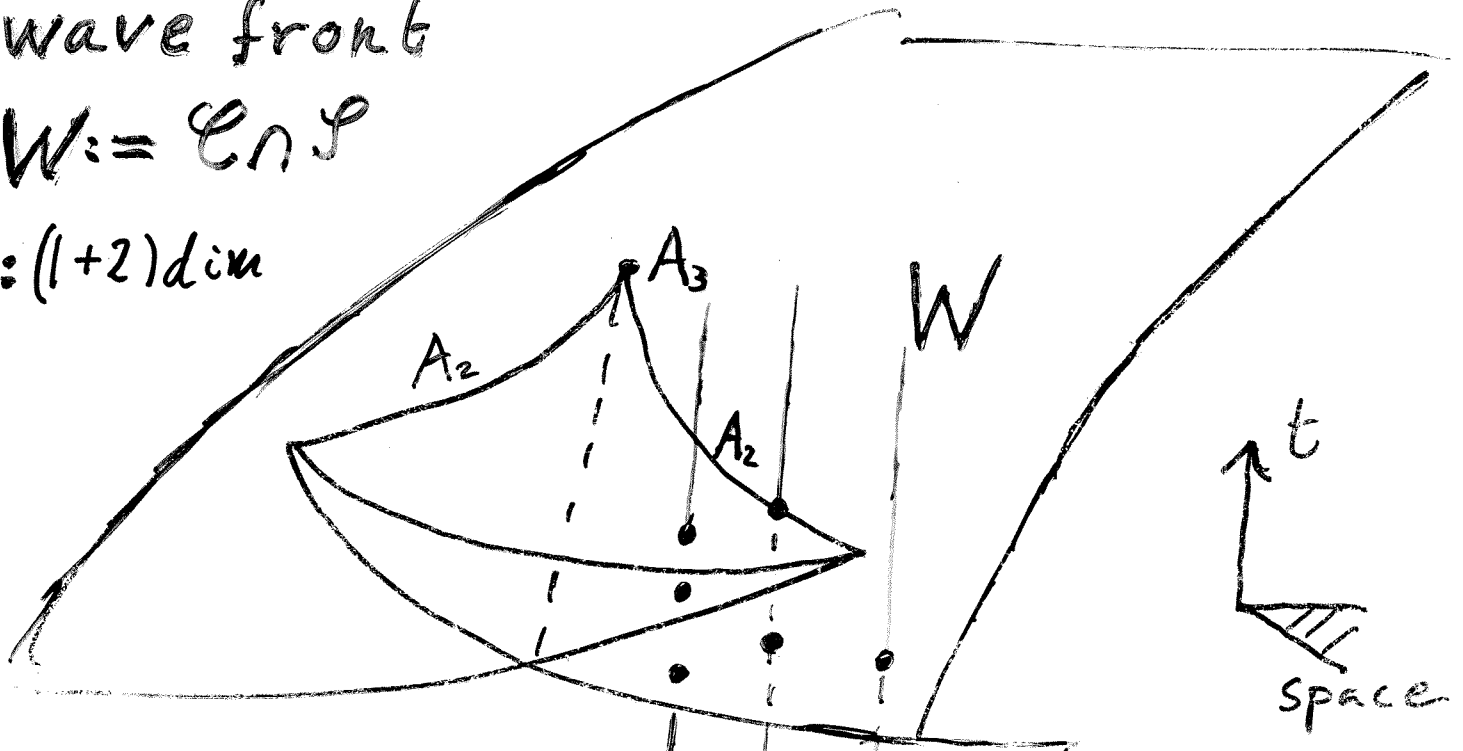
of emission of light reaching  $\mathcal{O}$ .

# Local shape of a (spacelike) 12

wave front

$$W := \mathcal{C} \cap \mathcal{S}$$

$$\mathcal{S} := (1+2)\text{dim}$$



Source positions:  
 $= \{\vec{y}\}$



Source world lines  $\leftrightarrow \vec{y}$

Points on  $W \leftrightarrow$  photon positions  
 $\leftrightarrow$  directions  $\vec{e}$  at  $\mathcal{O}$

Time of emission: function on  $W$

$$\vec{y} = l(\vec{e}), \quad t = T(\vec{e});$$

Source world lines and light rays  
 are transverse to  $W$

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Einstein field equation,  
( $M, g_{\alpha\beta}, U^{\alpha}, S, p$ ), dynamical  
cosmol. model

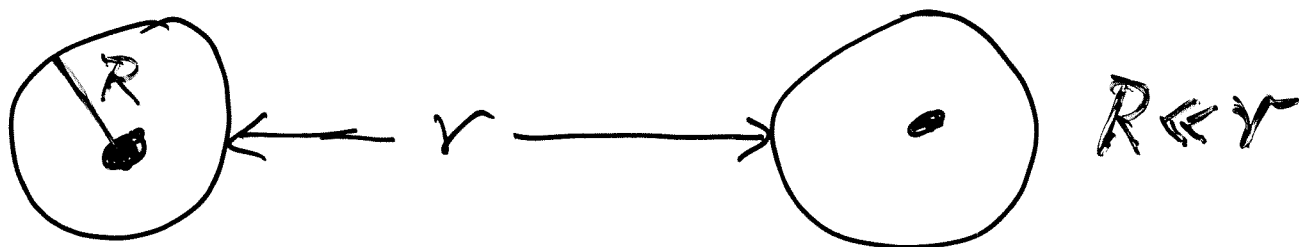
Mass distribution  $\leftrightarrow$   $g_{\alpha\beta}$   
 $\updownarrow$   
measur. relations

Zepelers: "Redshift" solution for  
lumpy universe model.

Way out: Newtonly perturbed Fried-  
mann model:

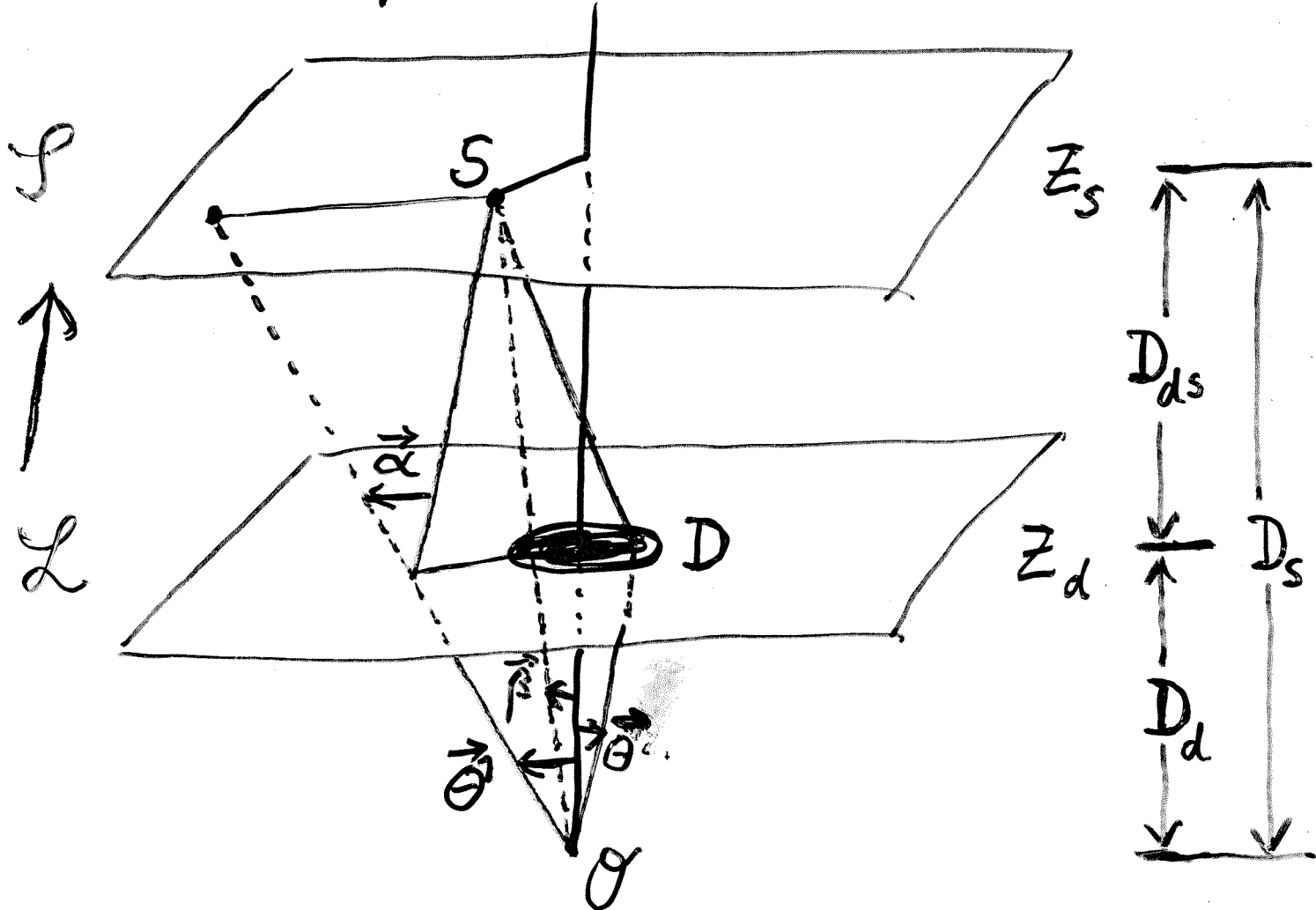
$$ds^2 = R^2[t] \left\{ -(1+2U)dt^2 + (1-2U)d\sigma_{\mathbb{R}^3}^2 \right\}$$

$|U| \ll 1, \partial_t U = 0, \{u\}$  static;



Nearly Newtonian regions

14 The approximate lens map  
 (for a single, geometr. thin,  
 transparent "lens")



$\vec{\alpha}$  : deflection angle  
 $\vec{\beta}$  : unperturbed } source  
 $\vec{\theta}^{(i)}$  : perturbed } position(s)  
 ( $\vec{\alpha}$  only observable)  
 $D$  : area distances  
 w.r.t. background metric

# ①5 The lens mapping and the time delay

Mass distribution in deflecting lens

$$\rightarrow \text{potential } \Psi(\vec{\theta}) = \Theta_E^2 \int \frac{dM'}{M} \ln|\vec{\theta} - \vec{\theta}'|$$

$$\rightarrow \text{bending angle } \vec{\alpha} = \frac{D_S}{D_{DS}} \vec{\nabla} \Psi$$

$\rightarrow$  lens map  $\mathcal{L} \rightarrow \mathcal{I}$

$$\underline{\vec{\beta} = \vec{\theta} - \vec{\nabla} \Psi(\vec{\theta})}$$

$$\Theta_E^2 = \frac{2m D_{DS}}{D_D D_S} = \frac{2m H_0}{c} \chi(z_D, z_S, \Omega_M, \Omega_\Lambda, \dots)$$

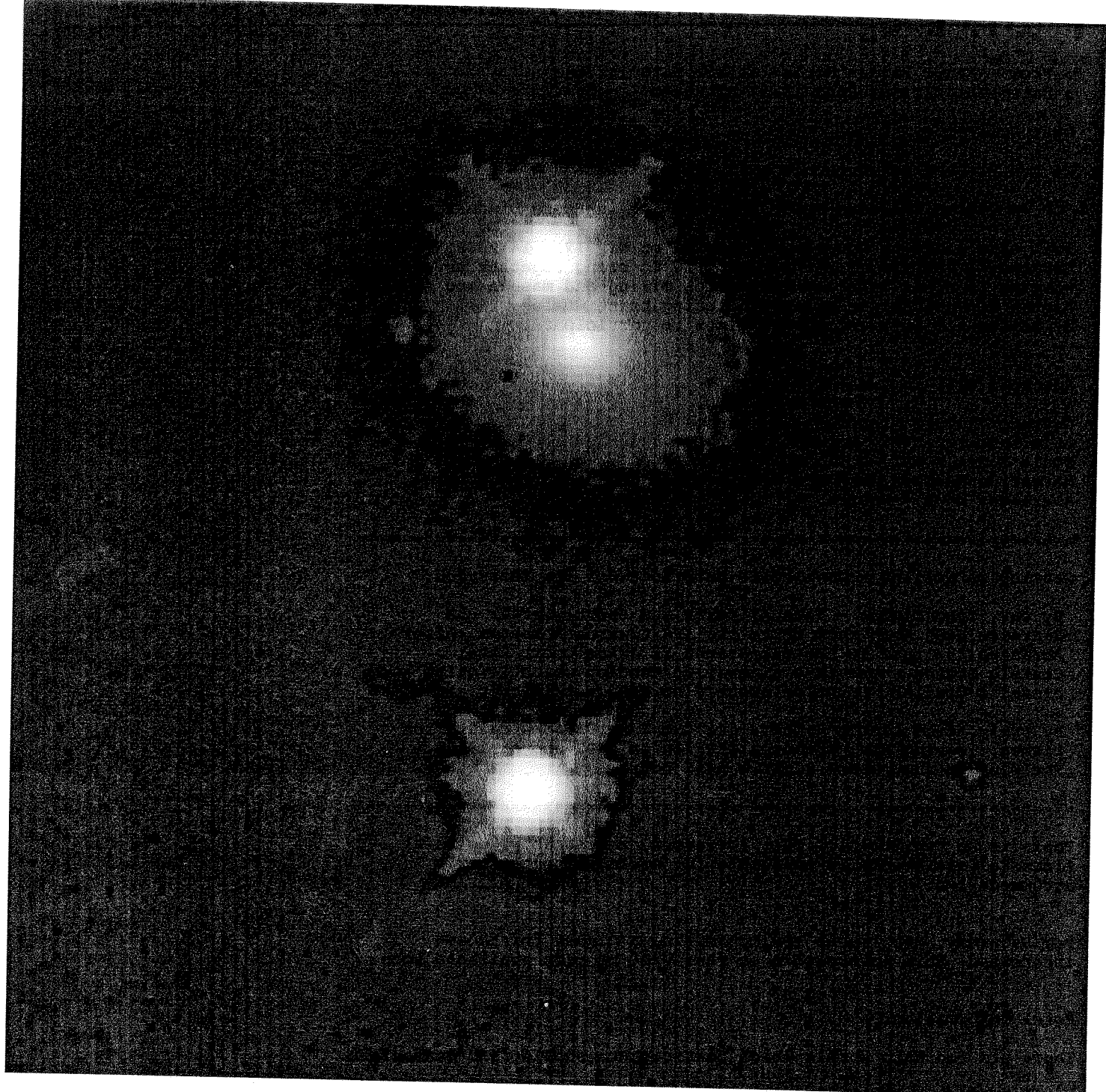
Cosmology

time delays:  $\phi = \frac{1}{2} (\vec{\nabla} \Psi)^2 - \Psi$

$$c \Delta t_{ij} = \frac{2m}{\Theta_E^2} (1+z_D) \cdot \{ \phi(\vec{\theta}_i) - \phi(\vec{\theta}_j) \}$$

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# Doppelquasar QO957+561



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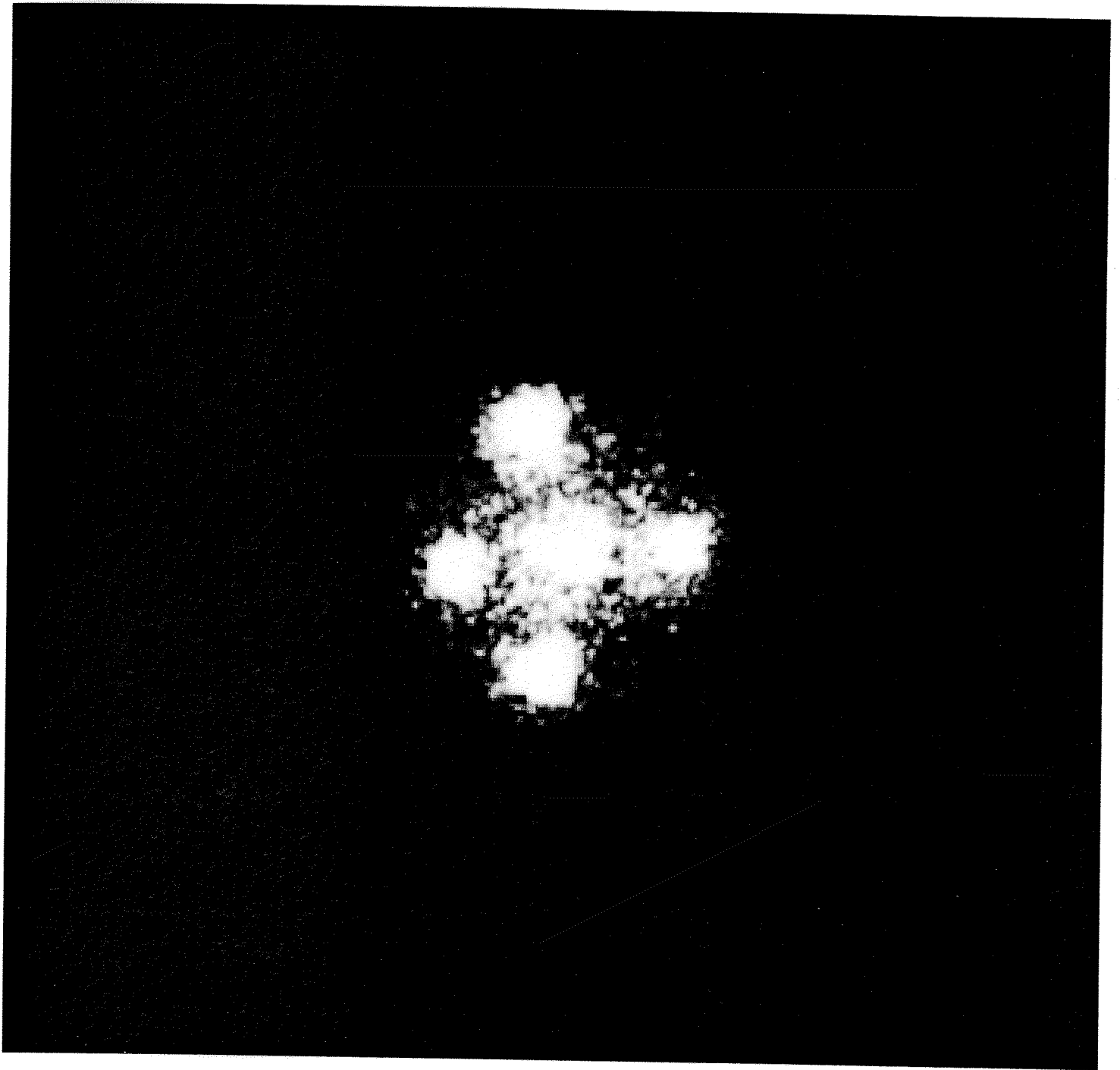
# Einsteinkreuz



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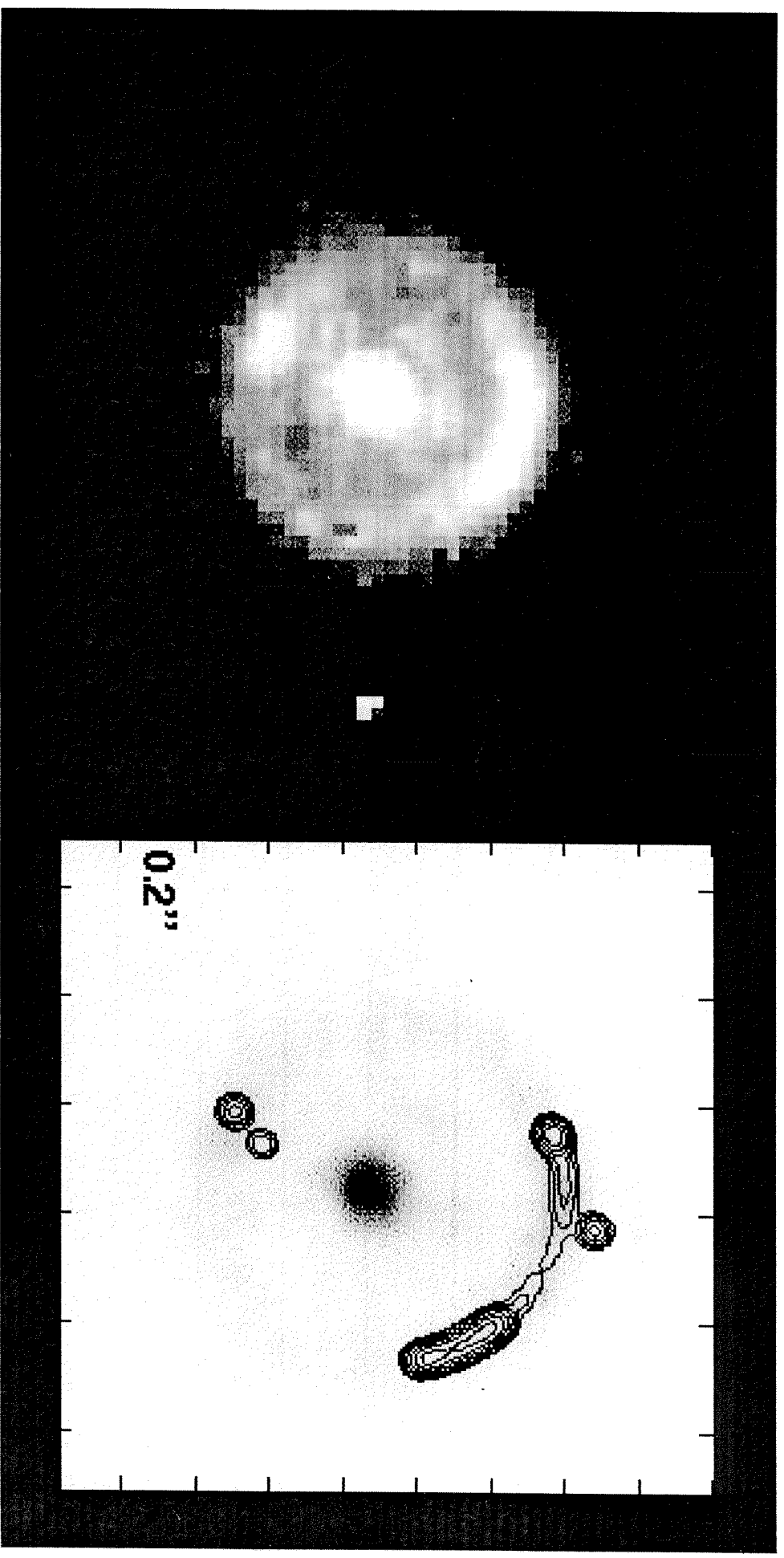
# Kleeblatt, H1413+117



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# Einsteinring 1938+666

links: Infrarot

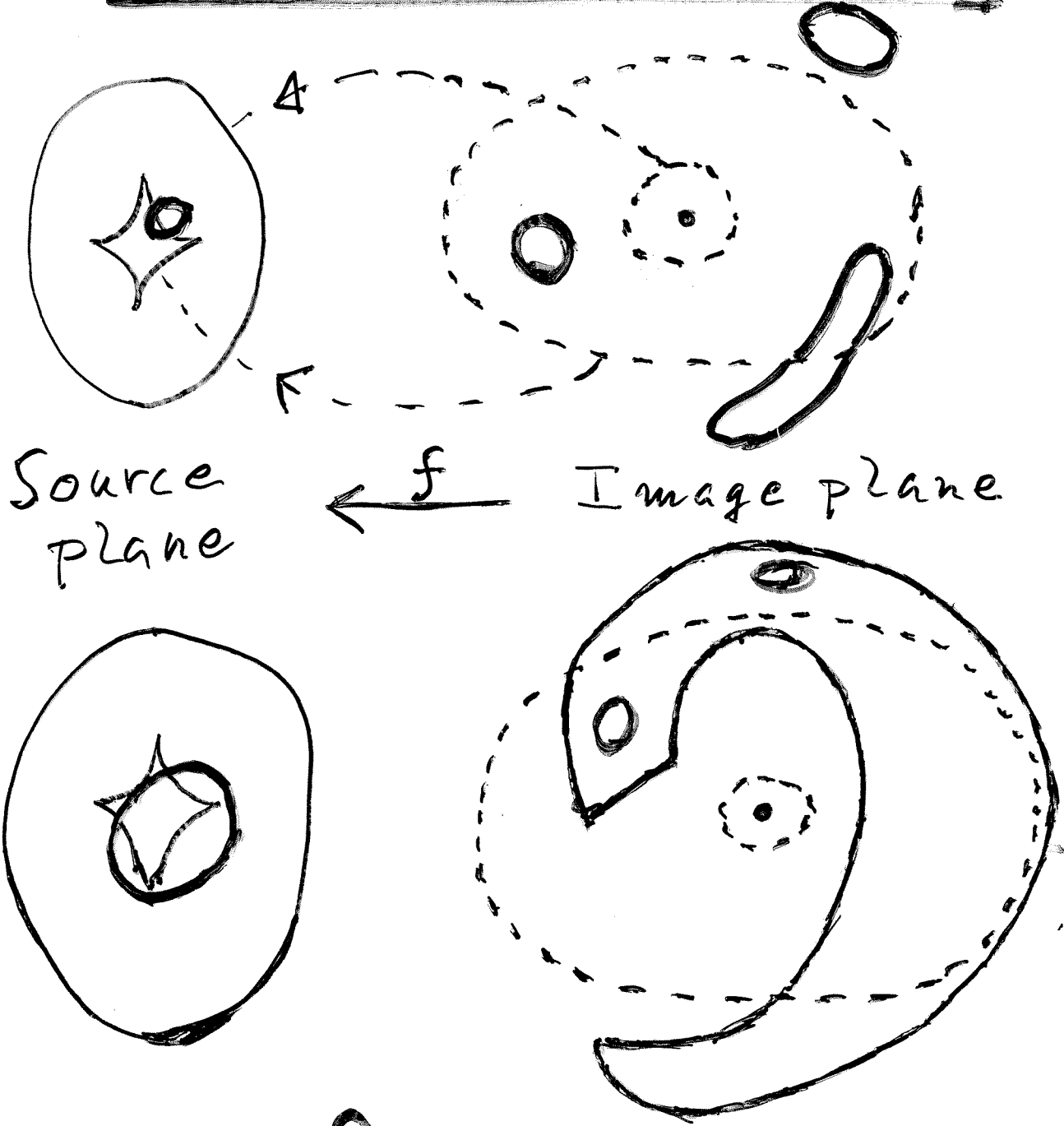
rechts: Radiokonturen



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# Elliptical "lens"



Source plane

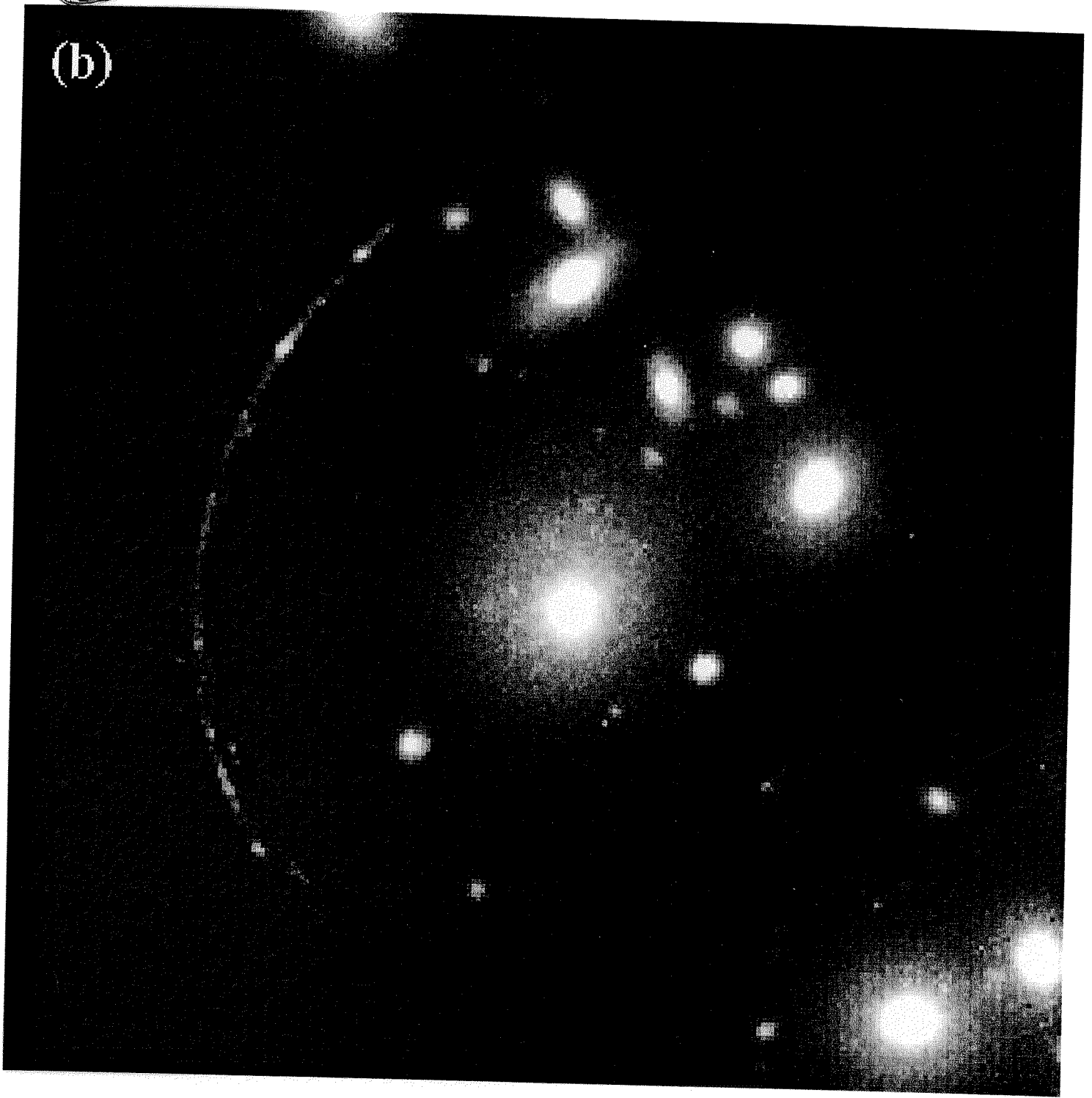
$f$  Image plane

Sources 0  
 Caustics |

Images |  
 Critical  
 curves |

| lens map  $f$ :  $\leftarrow$  one-valued  
 | physical "map":  $\rightarrow$  multi-valued

(b)

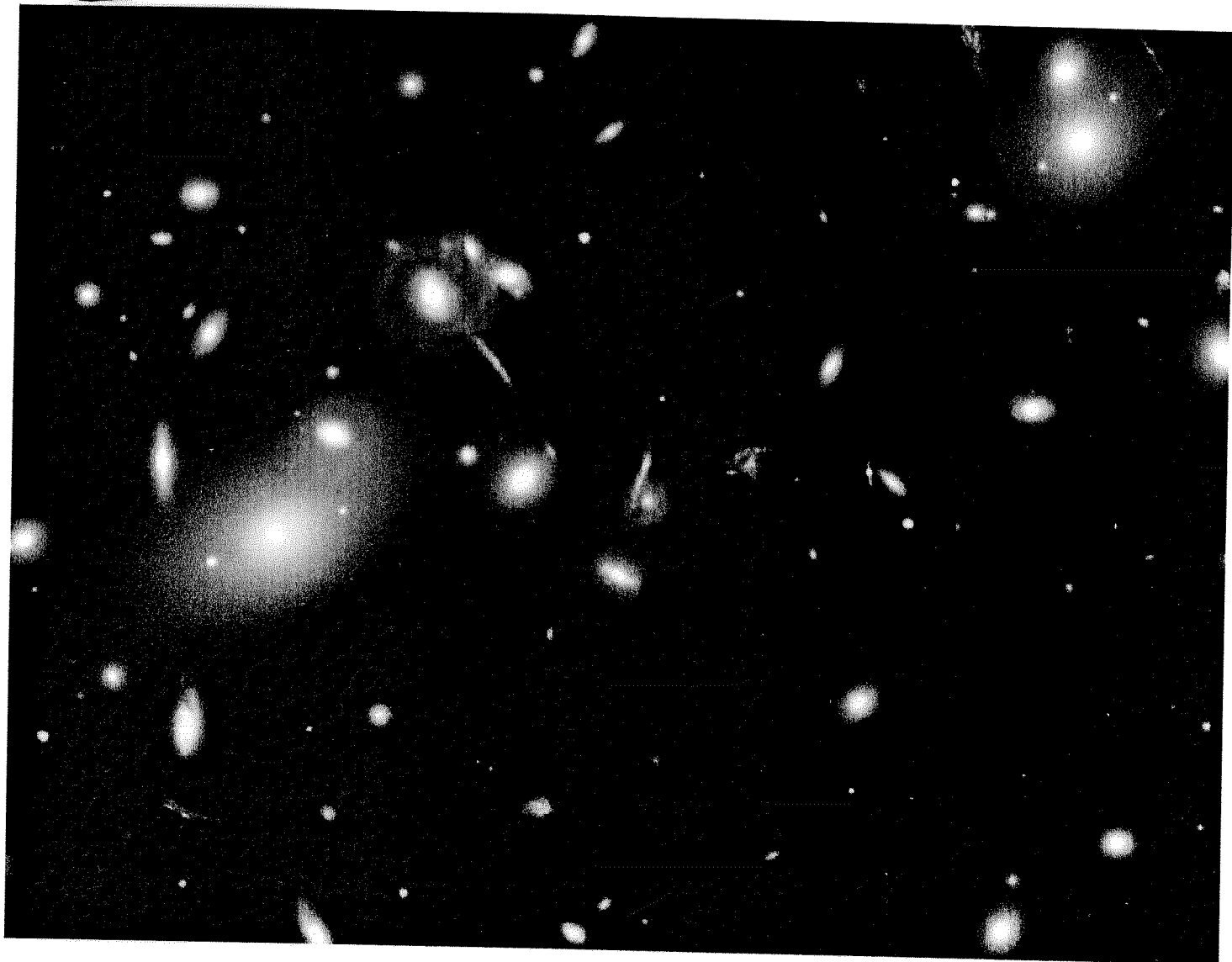


## Bogen im Galaxienhaufen Cl2244



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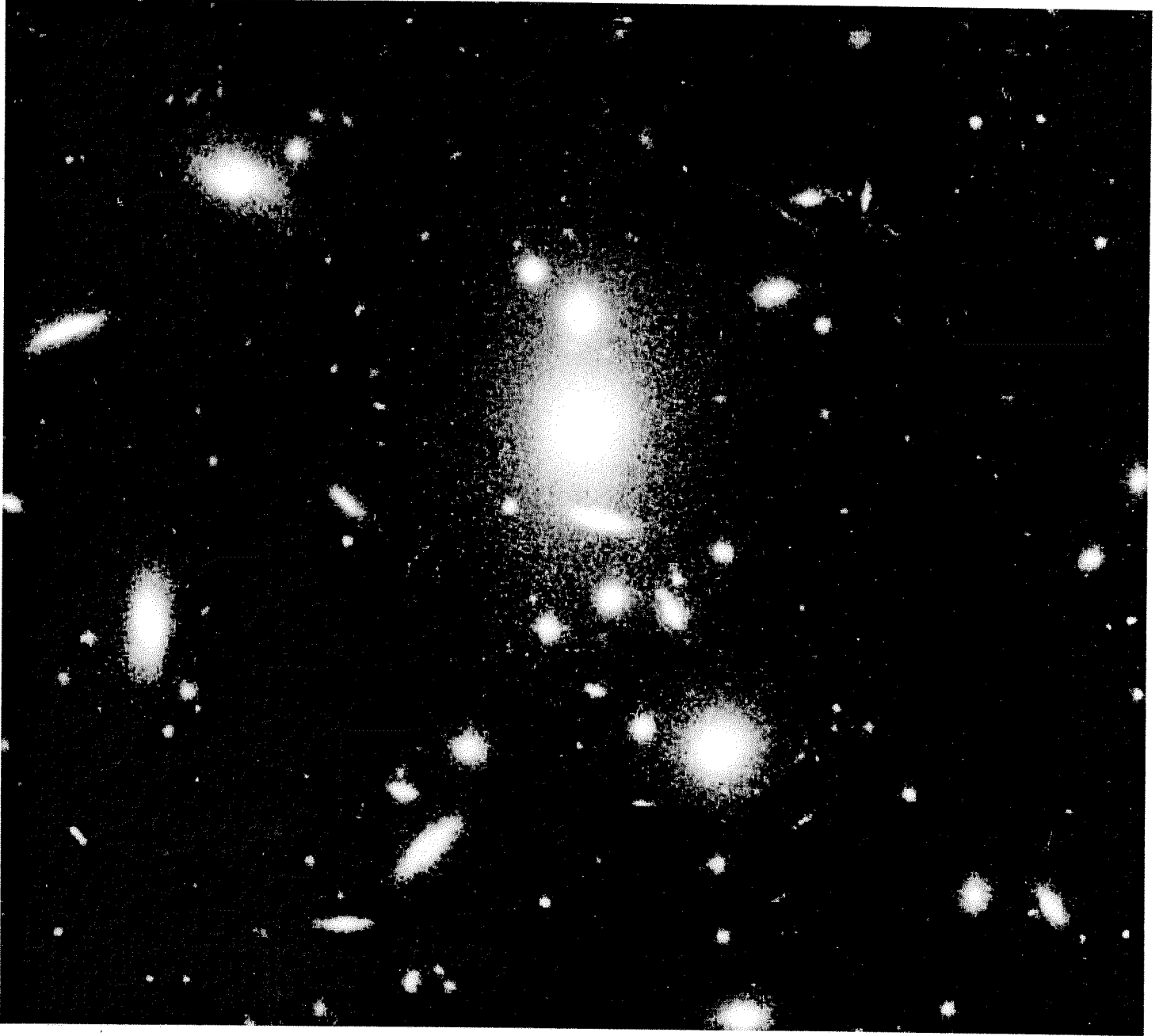


# Bögen in einem Galaxienhaufen



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# Galaxienhaufen CH358+62



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