Numerical construction of spacetimes

L. Lehner (UBC→LSU): ‘Introduction & questions’

D. Garfinkle (Oakland) : ‘Very local’ simulations

J. Frauendiener (Tuebingen) : ‘Spacetimes on the large’

L. Rezzolla (SISSA) : ‘Non-vacuum spacetimes’

NOTE: Dinner takes place on Thursday at 19:30 on the Institute grounds …. (and ants)

NR…why

If we think hard enough we won't need a computer

With the right resources we can simulate situations we can't even begin to think through, and thereby provide us with

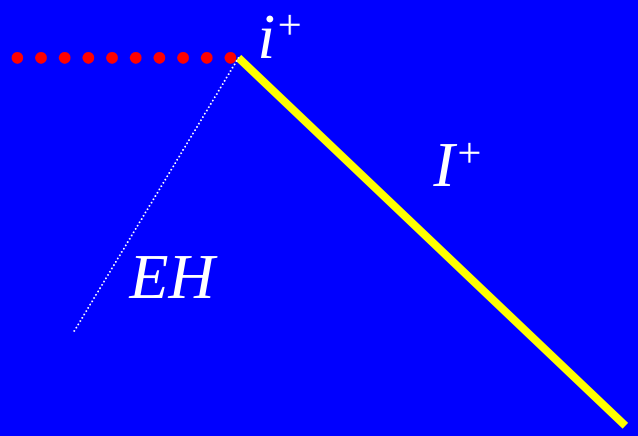
***completely new and** perhaps **unexpected** things to think about*

- Support?
  - Critical Phenomena in GR; (1D) [Choptuik]
  - Cauchy Horizons in charged/rot BH's. (1D) [Israel-Poisson, Piran,Brady,etc..]
  - Cosmological scenarios [Berger,Weaver,Hern,Madden,etc.etc….]
  - Wave-maps
  - KG field on bh spacetimes (AE=15/7; NE=3) [too many to list]

Promises…

- Strongly gravitating/highly dynamical spacetimes
  - GW detection, GW astronomy
  - Critical behavior, singularity structure
  - astrophysically relevant systems (AGN, GRB's….)

- 'Spacetimes in the large'
  - Is there a regular I^+,i^+ ?
  - How about peeling?
  - No hair?
  - How about that cosmic censorhip?



- 'Help' out in the search for quantum gravity?
  - Remove too much hand-waving (linear analysis + entropy arguments)

Aspects involved

- Solve $G_{\mu\nu} = \kappa T_{\mu\nu}$ through simulations
  - *'evolution' (reduction?)*
  - Initial and boundary data
  - *Singularities*
  - Coordinate issues
- Non-vacuum…
  - Fluid?, shocks (and all that…)
  - Initial and boundary data

+

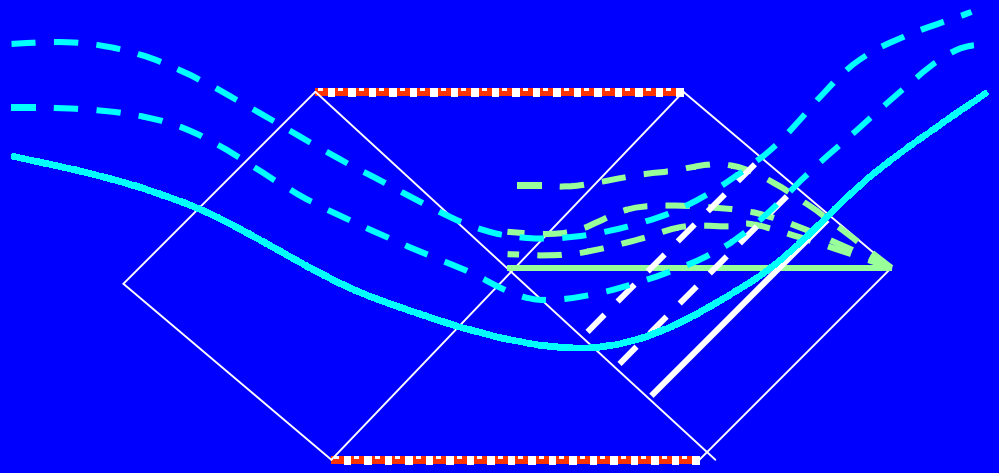
Computat Resources:

ALGORITHM ISSUES…

(too?) Many options! One size doesn't fit all

Options…

- Approaches and particular issues
  - ***3+1***; ***characteristic*** and ***Conformal Einstein*** approaches



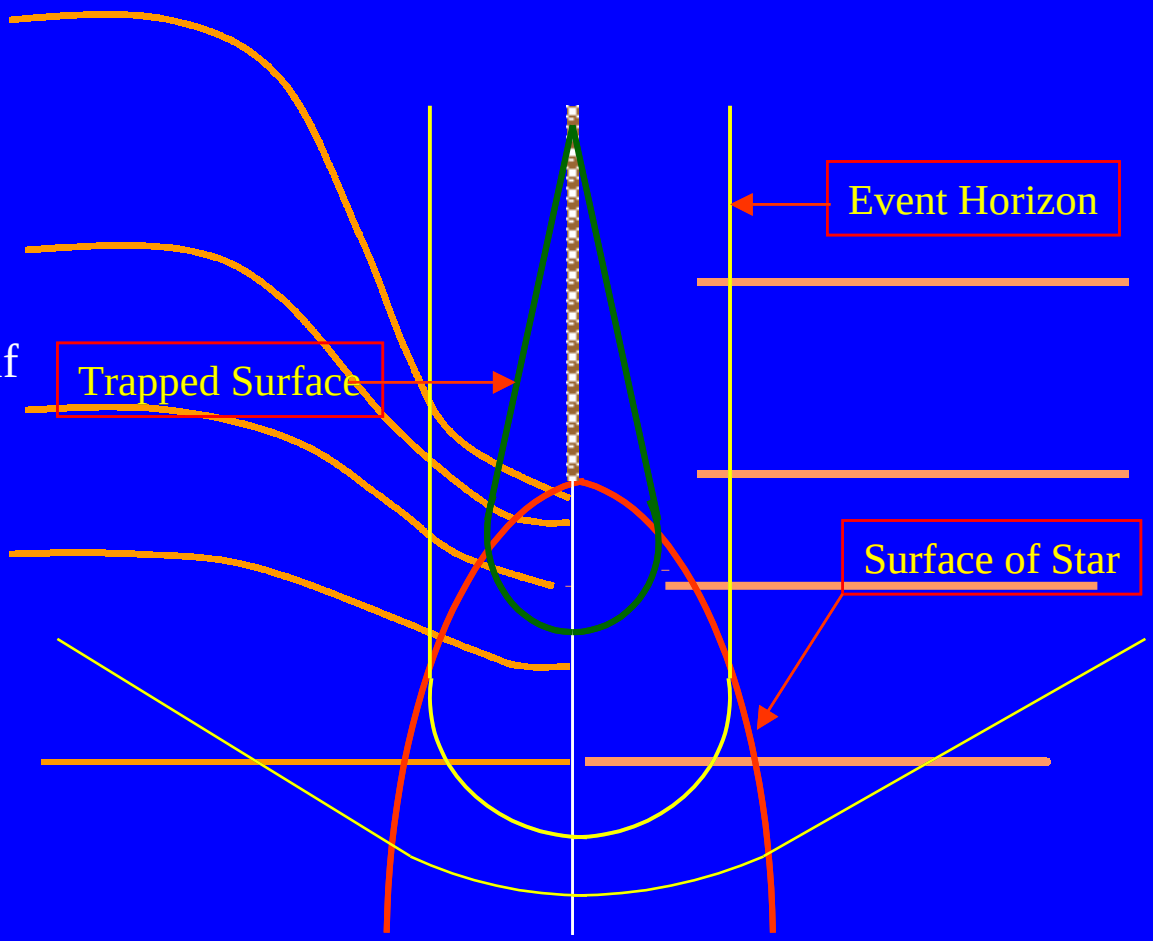
• 3+1: general, flexible gauge, timelike outer boundary, most popular.

• characteristic: more restrictive, rigid gauge, null outer boundary (at future null infinity), when it can be applied works ‘scarily’ well

• conformal: general, flexible gauge, outer boundary hidden by null infinity, can accommodate for the 2 previous, ID and gauge more involved.

Singularities…

- ‘Tricky’ to handle analytically
- Computational nightmare!
- What to do?
  - Avoid
  - Excise (in principle, OK, if cosmological censorship)



(some) Current problems

- Foundation/funding problem: Binary systems
  (note: 2/3 orders of magnitude short of computational power for 'confortable' 3D simulations)
- Critical phenomena in axisymmetry.
- Cosmology
- Spacetimes on the large
- Higher dimensional gravity

Advances in all of these,
in most cases painful efforts involved

We need your help!

Reduction/formulation…

Boundary conditions (IBVP)

Initial data

Gauge issues

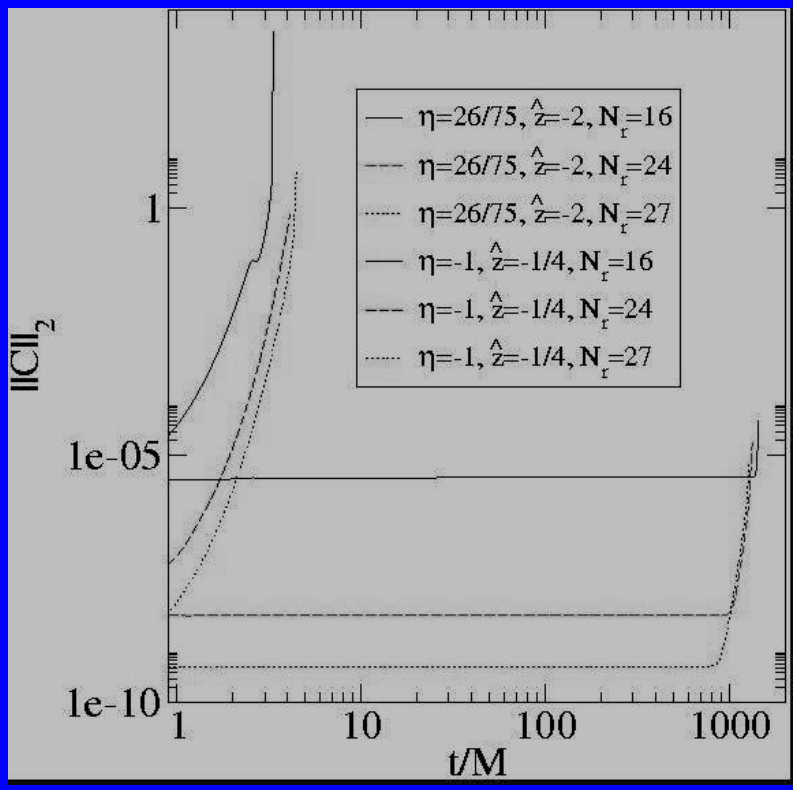
Reduction/Formulation:
'well posedness is not enough'

- Well posedness does not rule out exponential growth, but we would like so. *[especially in the Cauchy problem]*
  - Initial data only up to round-off (at best…)
  - Complicated by gauge issues
  - Driven by boundary conditions

Constraint eqns unique, but not so the evolution eqns. Can we take advantage of this?

Practical *(typical)* example…

- Generalized ‘Einstein-Christoffel’ reduction [Anderson-York; *Kidder-Scheel-Teukolsky*]
- 2-parameter family of reductions. *Same principal part only different in lower order terms*
- Growth from energy estimates in *rough agreement* with observed behavior (Lindblom-Scheel)
- *Qn 1: Is there a way to obtain sharp growth estimates which involve ‘background’ and gauge?*
- *Qn 2: Is it possible to single out preferred reductions?*
- *Qn 3: Alternatively, if we use the constraints (and deal with an elliptic-hyperbolic system), is the expected behavior any better?* (inner boundary conditions?!)



IBVP

- 50 yrs of the Cauchy IVP (Choquet Bruhat), just a few of the IVBP (Friedrich-Nagy).
  - Dissipative boundaries theory.

- 'Traditional' handling: use 'physical arguments' to define values for all variables. (unlikely consistent in general…we deal with a constrained system)

- Road to the cure (A): [LSU-FaMAF,Pitt-AEI]
  - 1st: give just to incoming modes (easy)
  - 2nd: constraints restriction on incoming modes (hard)
  - 3rd: give the free modes controlling the incoming radiation (??)

- Road to the cure (B): [Caltech-Cornell,LSU-FaMAF]
  - 1st and 2nd of (A)
  - (almost) Compactify (Conformal Einstein eqns and Characteristic formulation OK, standard Cauchy problem?) [UBC]

- Road to the cure (C): [AEI,UIUC,WashU,PSU,UBC,…]
  - Systematic search for consistent boundary conditions, checked by stability and accuracy of the solution. (standard approach).

Issues…

- Well posednes beyond the ‘quarter problem’
  - Constrained boundary conditions lead to a system ‘living’ at the boundary

- Radiation control require:
  - higher derivatives formulation: (Calabrese-Pullin-Reula-Sarbach-Tiglio; Schmidt-Szilagyi-Winicour)
  - ‘conformally hidden’ boundaries
  - ‘live’ with it making sure it doesn’t bother. Needs boundaries far out (and knowing behavior of fields)

- *Qn 4: Can we control the behavior at ‘corners and edges’ for GR?*
- *Qn 5: Can we compactify (in 3D) the 3+1 ‘standard’ approach?*

Initial data

- No problems giving consistent initial data
  - Traditional, gluing, etc.
  - 'create' the initial data with 'matter/GW' collapsing sources (not too much explored, but quite natural)

- Free of spurious radiation/correct physics.
  - 'One' slice won't do (unless a proper matching to a previous 'stage' is performed consistently)
  - More than one (thin sandwich) better control but likely not enough.
  - 'Flush' out of radiation is a way out (assuming initial physical stage is appropriate). Requires evolution (and hence more than one slice)

Qn 6: Can we get a better handling of the radiation content?

Qn 7: If 'matching' is a way out (say PN expansion), how to do it?

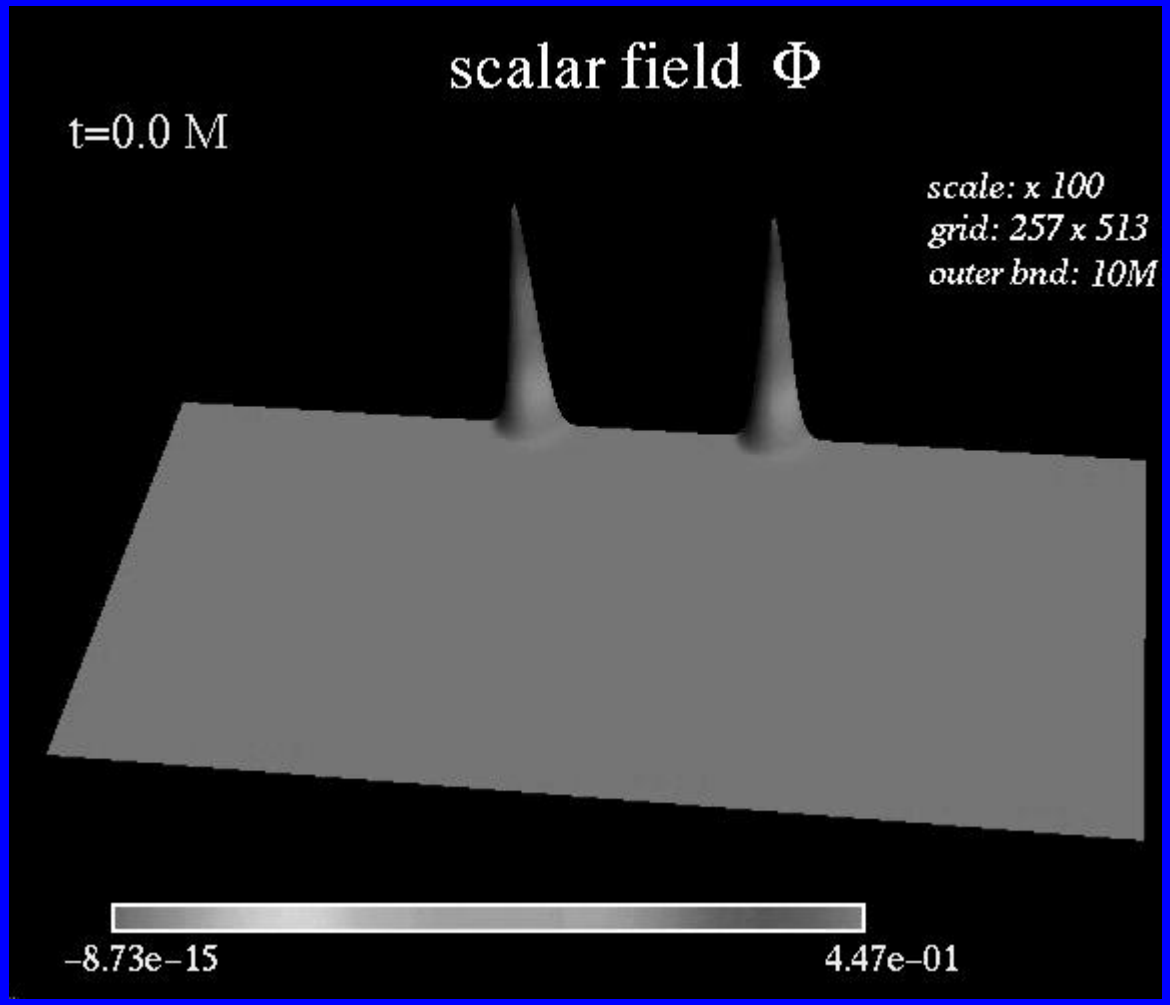
(some) Gauge issues

- Basic issues:
  - K=0 might not be the most convenient when dealing with singularities.
  - Need slices ‘penetrating’ the horizon.
  - Combinations of K_{ij} modulating some exponential modes
- Physical issues:
  - Appropriate for stationary behavior
  - Adapted for (approx) killing fields

Qn 8: How to do all this (and more!) ?

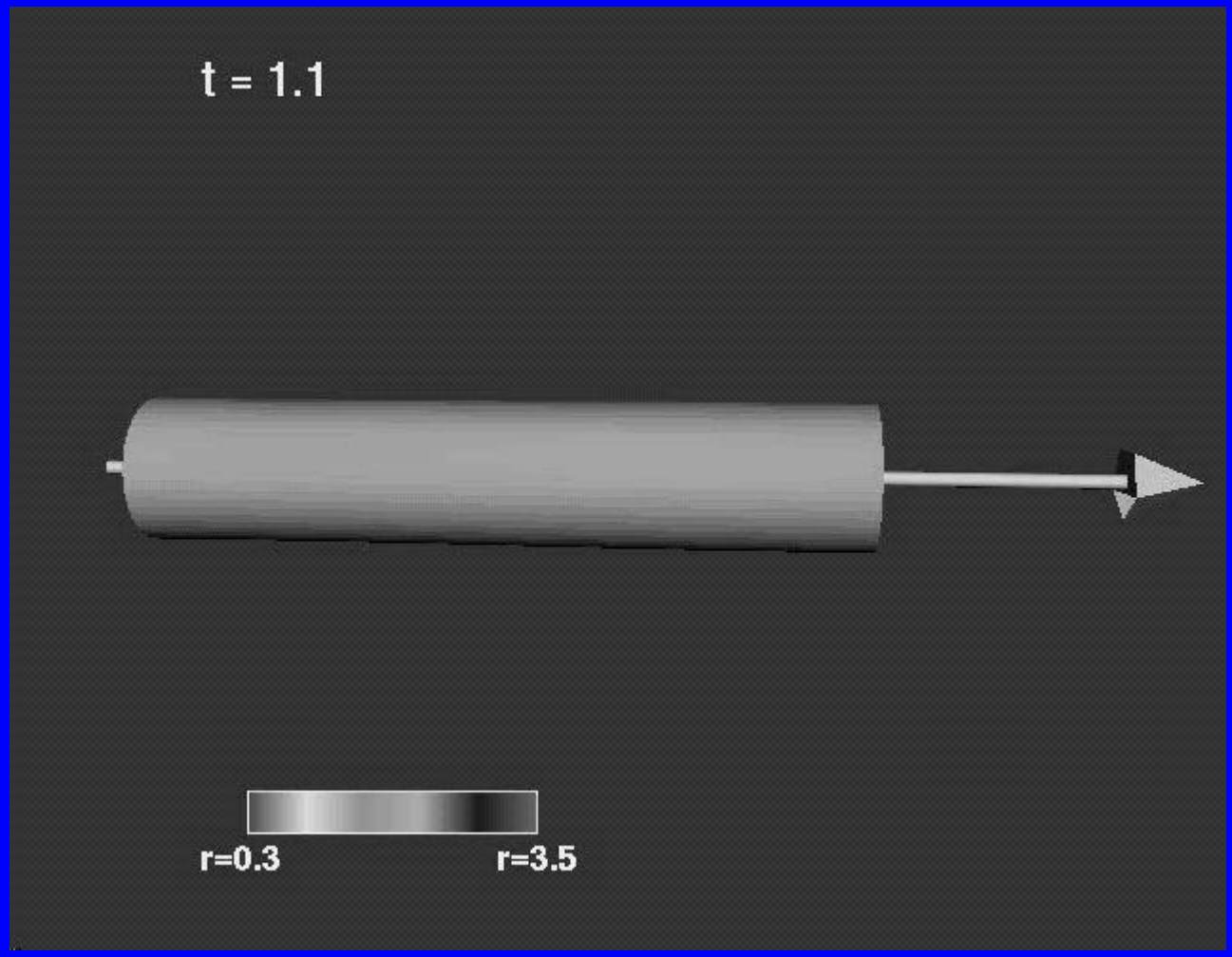
Trailers….

Mass-less KG field coupled to gravity

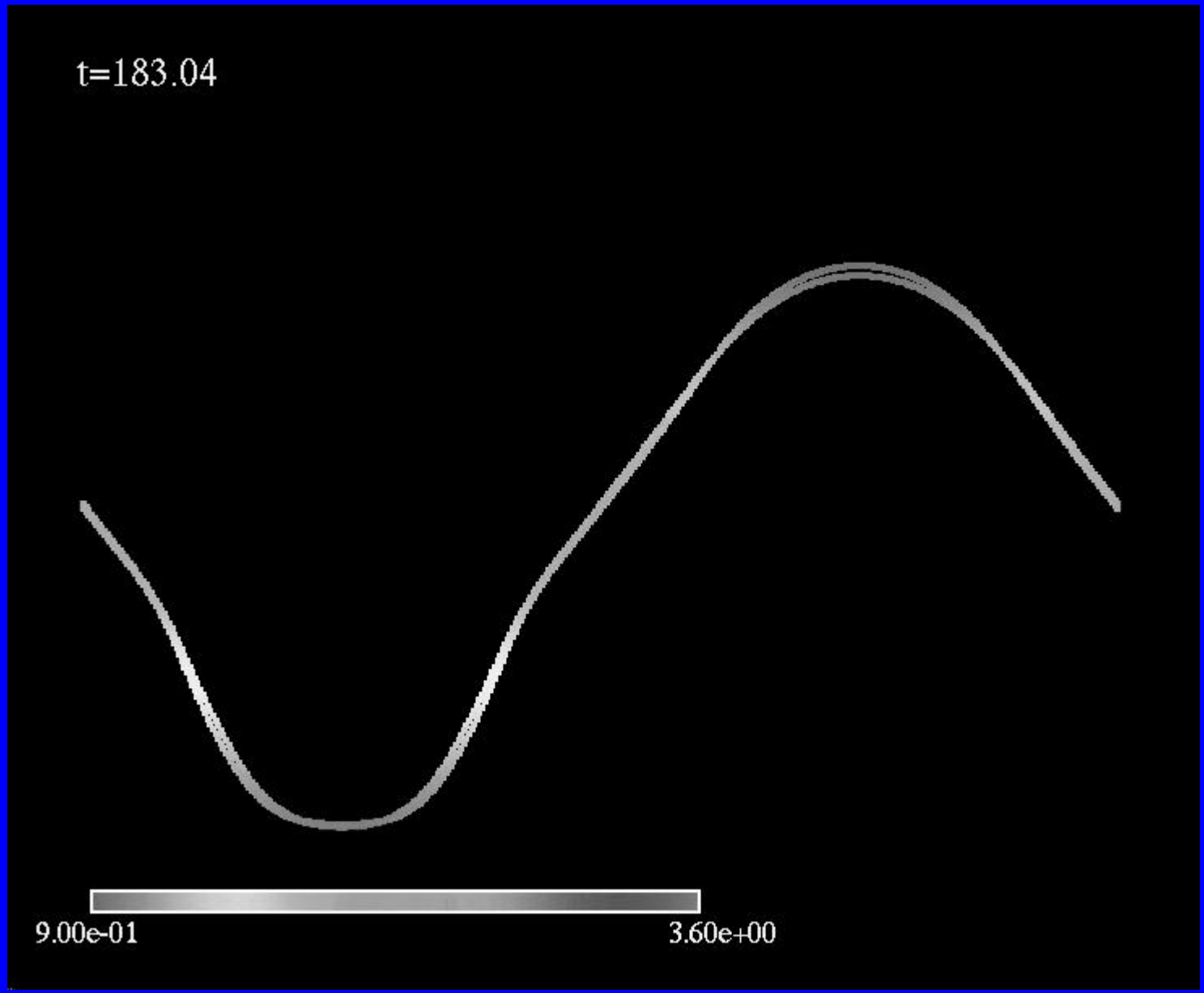


Choptuik, Hirshman, Liebling, Pretorius

Black string ‘instability’ ?



Choptuik, L.L, Olabarrieta, Petryk, Pretorius, Villegas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